

# Prediction of Federal Funds Target Rate:

*A Dynamic Logistic Bayesian Model Averaging Approach*

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# AGENDA

## *Research motivation*

1. Introduction
2. Methodology & Simulation Exercises
3. Data & target rate characteristics
4. Empirical results: Estimation & Forecasting
5. Conclusions
6. Future research endeavors

# RESEARCH MOTIVATION

- For decades the fields of finance and macroeconomics dealt with interest rates, asset prices and the yield curve in a total different way and without much interaction. As **Diebold and Rudebusch (2013)** point out in their book titled “*Yield Curve Modeling and Forecasting*”

*“**In macro models**, the entire financial sector is often represented by a single interest rate with no accounting for credit or liquidity risk and no role for financial intermediation or financial frictions. Similarly, **finance models** often focus on the consistency of asset prices across markets with little regard for underlying macroeconomic fundamentals. To understand important aspects of the recent financial crisis **a joint macro-finance perspective** is likely necessary”.*

- Although extensive research have been conducted regarding the predictability of macroeconomic variables based on the dynamics of the term structure of interest rates; there is a lack of empirical evidence focusing on the **ex-ante signals and predictive power of FRAs derived from Overnight-Indexed Swaps (OIS)**, in order to anticipate the path of future monetary policy.

# 1. INTRODUCTION

- **Market expectations** about the expected path of central bank's target repo rate can have important consequences for financial markets and the economy as a whole.
- **Expectations could be derived from** *fed funds futures contracts, forward rate agreements (FRAs), overnight-indexed swaps (OIS), Eurodollar futures, and options on interest rate futures.*
- **Interest rate derivatives (IRDs)** enable market participants to hedge against or speculate on potential movements in short-term interest rates, as a result, **IRDs are a rich and timely source of information about market expectations.**



The **federal funds rate** is the interest rate at which depository institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight.

# 1. INTRODUCTION (CONT.)

- **Gürkaynak, R. (2005)** uses long-maturity federal funds futures contracts to extract policy expectations and surprises at horizons defined by future FOMC meetings. In a recent study, **Crump et al (2014)** present evidence about how the paths of the policy rate constructed from fed funds futures, OIS, and Eurodollar futures are useful tools to analyze market expectations.
- **Estrella and Hardouvelis (1991), Estrella and Mishkin (1998), Stock and Watson (2000), Ang and Piazzesi (2003), and Diebold et al (2006)**, find strong evidence of macroeconomic effects on the future yield curve.
- Although, extensive research have been conducted regarding the predictability of macroeconomic variables based on the dynamics of the term structure of interest rates; there is a **lack of empirical evidence focusing on the ex-ante signals and predictive power of FRAs** estimated from Overnight-indexed Swaps (OIS), in order to anticipate the path of future monetary policy.
- **S. van den Hauwe et al (2013)**, develop a Bayesian framework to model the direction of FOMC target rate decisions. Most predictive ability is found for, first, economic activity measures like industrial production, the output gap and the coincident index, and, second, term structure variables like interest rate spreads.

## 2. METHODOLOGY

the econometric framework used to perform statistical inference regarding the target repo rate decisions made by Central Banks is the following:

- First of all, we set  $r_t$  as the prevailing rate at the end of the month  $t$ ,  $t = 1, 2, \dots, T$  being  $T$  the sample size, and  $\Delta r_t = r_t - r_{t-1}$ , the variation of the rate. Then, as our main objectives are to find the determinants and predict **upward** ( $\Delta r_t > 0$ ) and **downward** ( $\Delta r_t < 0$ ) movements in the repo rate, we adopt the following definitions:

$$y_t^u = \begin{cases} 1 & \text{if } \Delta r_t > 0 \\ 0 & \text{if } \Delta r_t = 0, \Delta r_t < 0 \end{cases}$$

$$y_t^d = \begin{cases} 1 & \text{if } \Delta r_t < 0 \\ 0 & \text{if } \Delta r_t = 0, \Delta r_t > 0 \end{cases}$$

- To estimate our models, we extend the **dynamic model averaging procedure for dynamic logistic regressions** developed by [McCormick et al. \(2012\)](#). In particular, we implement a **Markov Chain Monte Carlo Model Composition procedure**, or **MC<sup>3</sup>**, for model selection. This adaptation reduces enormously the computational burden of the algorithm.

## 2. METHODOLOGY (CONT.)

- We propose a **MC<sup>3</sup> algorithm** that goes over the model space looking for the best models reducing drastically the computational burden. Therefore, we try to find the best “active” subset of the models at each time.
- This econometric approach takes into consideration simultaneously three (3) desirable statistical characteristics: **(i) dynamic parameters**, **(ii) dynamic Bayesian Model Averaging**, and **(iii) an autotuning procedure**, all based on the best models.
- The choice of the **BMA methodology** is based on the fact that this framework is firmly grounded on statistical theory following the rules of probability. It minimizes the sum of Type I and Type II error probabilities; its posterior point estimates minimize the mean square error, and its posterior predictive distributions perform better relative to other estimators (Raftery and Zheng, 2003).
- The **MC<sup>3</sup>** procedure is an algorithm for drawing candidate models over the space  $\mathcal{M}$ , based on a **Metropolis-Hastings algorithm** (Metropolis et al., 1953; Hastings, 1970). It simulates a chain of models,  $M^{(k)}$  (**for**  $k = 1, 2, \dots, S$ ), where the mechanism samples candidate models from a particular distribution, and accepts them with a probability. If a candidate model is not accepted, the chain remains in the current model (Koop, 2003).

## 2. METHODOLOGY (CONT.)

- In particular, we initially build a design matrix  $\mathbf{X}_{J \times K}$ , selecting predictors using a Bernoulli distribution with probability  $\mathbf{p}$ . Such that each row of  $\mathbf{X}$  defines a candidate model, and our goal is to find the  $J$  best models.
- We calculate the **average posterior model probability** for these initial models,  $\pi(M_T^{(k)} | y_{1:T})^{Ave} = 1/T \sum_{t=1}^T \pi(M_t^{(k)} | y_{1:t})$ , and find the model that has the minimum posterior model probability,  $M_t^{(Min)}$ . Then, a candidate model  $M_t^{(c)}$  is drawn randomly from the set of all models excluding the initial models, and we estimate its **posterior model probability**.

We accept this candidate with probability: 
$$\alpha(M_T^{(Min)}, M_T^{(c)}) = \text{Min} \left\{ \frac{\pi(M_T^{(Min)} | y_{1:T})^{Ave}}{\pi(M_T^{(c)} | y_{1:T})^{Ave}}, 1 \right\}$$

## 2. SIMULATION EXERCISES

- The results of the Monte Carlo experiment in order to show **the ability of our Model Composition strategy to solve a variable selection problem** are the following: In particular, we evaluate the performance of the algorithm to detect the hidden data generating process (**d.g.p.**) using different number of iterations  $S = \{100, 500, 1000, 5000, 10000\}$ .

$$y_t^* = \beta_0 + \beta_{1t}x_{1t} + \beta_{2t}x_{2t} + \beta_{3t}x_{3t} + \beta_{4t}x_{4t} + \beta_{5t}x_{5t} + \epsilon_t$$

- The **data generating process** is given by  $\rightarrow$

$$y_t = \begin{cases} 1, & y_t^* > 0 \\ 0, & y_t^* \leq 0 \end{cases}$$

- Where  $x_{it} \sim^{i.i.d} \mathcal{N}(0, 1)$  and  $\epsilon_t \sim^{i.i.d} \mathcal{LG}(0, 1)$ ,  $i = 1, \dots, 5$ ,  $t = 1, 2, \dots, 5000$ .

The **d.g.p**  
changes  
through time

**Table 1: Conditions under which data were generated**

	$\beta_{0t}$	$\beta_{1t}$	$\beta_{2t}$	$\beta_{3t}$	$\beta_{4t}$	$\beta_{5t}$
$t = 1, \dots, 4000$	1	-2	-1	$1 + t/5000$	2.5	-1
$t = 4001, \dots, 5000$	1	0	-1	$1 + t/5000$	0	-2

## 2. SIMULATION EXERCISES (CONT.)

- The design matrix includes 9 additional regressors that are not part of the d.g.p., such that  $x_{it} \sim^{i.i.d} \mathcal{N}(0, 1)$ ,  $i = 6, \dots, 14$ ,  $t = 1, 2, \dots, 5000$ . In addition, we use as **training sample 60% of data**.
- This setting implies that there are  $2^{15}$  possible models, that is, **32,768 models**. Our goal is to find the 20 best models, and determine if these encompass the true data generating process
- We can see in **Table 2** the **Posterior Inclusion Probability (PIPs)** of each variable. Among the regressors that are part of the d.g.p.,  $x_{1t}$  has the minimum **PIP (0.65)** followed by  $x_{4t}$  (**0.70**) with **100 iterations**.
- Regarding  $x_{2t}$ ,  $x_{3t}$  and  $x_{5t}$  their PIPs increase as the number of iterations increase.

Table 2: Posterior Inclusion Probability

Variable	Iterations				
	100	500	1000	5000	10000
$x_{1t}$	0.65	0.85	0.75	0.75	0.75
$x_{2t}$	0.80	0.95	0.95	0.95	0.95
$x_{3t}$	0.95	1.00	0.95	0.95	1.00
$x_{4t}$	0.70	0.75	0.75	0.70	0.85
$x_{5t}$	0.75	0.90	0.95	1.00	0.95
$x_{6t}$	0.45	0.25	0.20	0.00	0.10
$x_{7t}$	0.50	0.40	0.20	0.20	0.20
$x_{8t}$	0.55	0.35	0.30	0.25	0.05
$x_{9t}$	0.45	0.35	0.50	0.20	0.20
$x_{10t}$	0.35	0.25	0.30	0.30	0.30
$x_{11t}$	0.60	0.45	0.25	0.00	0.10
$x_{12t}$	0.45	0.30	0.25	0.15	0.15
$x_{13t}$	0.35	0.45	0.45	0.40	0.45
$x_{14t}$	0.35	0.25	0.15	0.20	0.10

## 2. SIMULATION EXERCISES (CONT.)

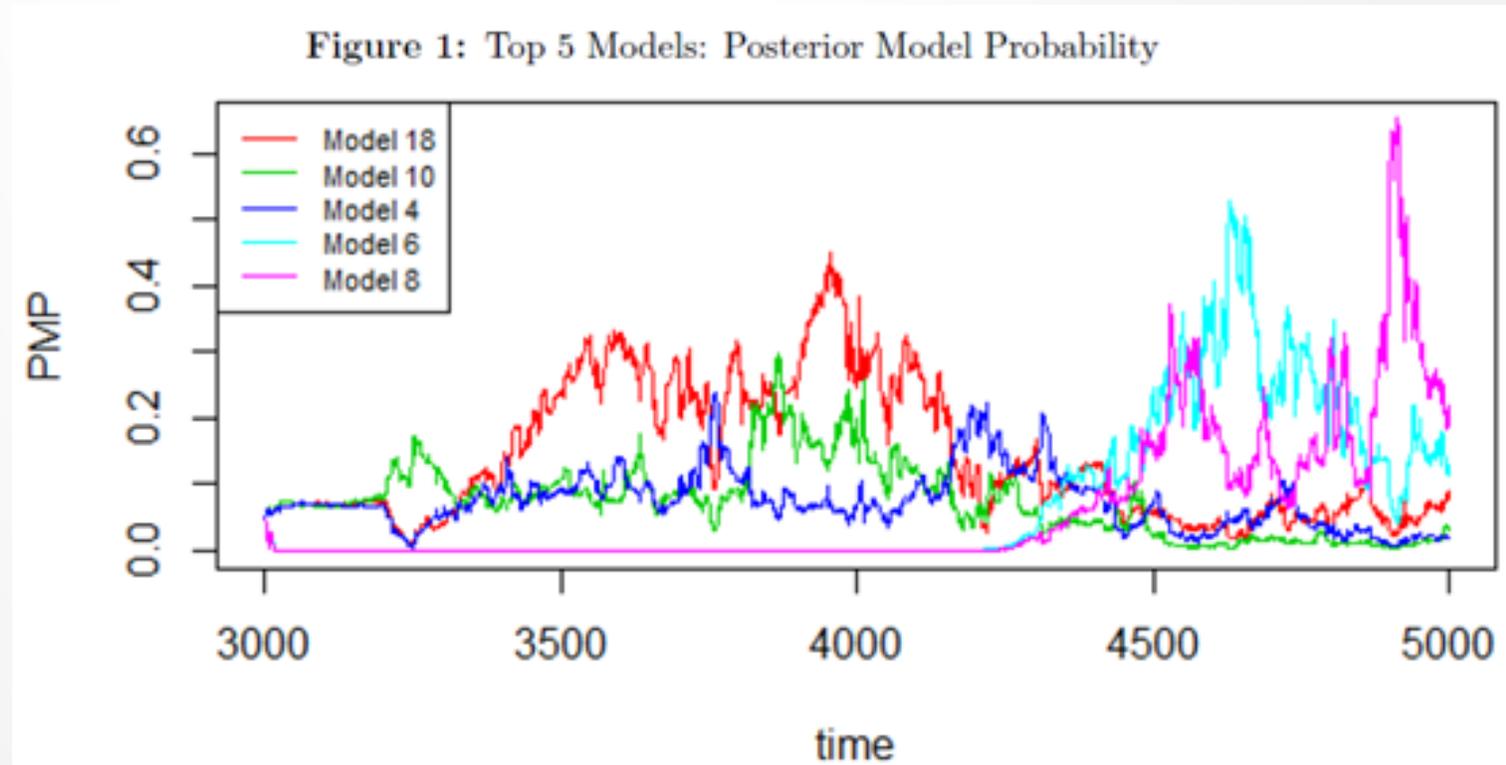
- Table 3** shows the **top 20 best models** of the simulation exercise with **10000 iterations**. We can see there that most of the models include the first five (5) regressors, which in turn are part of the d.g.p. **Models # 18, 10, 4, 6** and **8** have the highest average Posterior Model Probabilities (**PMPs**). **The average PMP are: 0.14, 0.08, 0.07, 0.07** and **0.06**, respectively.

**Table 3: Best Models: 10000 iterations**

Variable	Model																				PIP
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
$x_{1t}$	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0.75
$x_{2t}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0.95
$x_{3t}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1.00
$x_{4t}$	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0.85
$x_{5t}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0.95
$x_{6t}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0.10
$x_{7t}$	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0.20
$x_{8t}$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0.05
$x_{9t}$	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0.20
$x_{10t}$	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	0.30
$x_{11t}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0.10
$x_{12t}$	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0.15
$x_{13t}$	0	1	1	1	1	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0.45
$x_{14t}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0.10
Av. PMP	0.04	0.04	0.04	0.07	0.04	0.07	0.05	0.06	0.04	0.08	0.04	0.03	0.04	0.06	0.04	0.04	0.04	0.14	0.05	0.00	

## 2. SIMULATION EXERCISES (CONT.)

- **Figure 1** depicts the **PMP** for the **top 5 models**. **Model 18**, which is the true d.g.p. in the first sub-sample, has the highest PMP in this data subset.
- In the second subset, **Model 6** has the highest PMP followed by model **8**. Those models exclude variables  $x_{1t}$  and  $x_{4t}$ , which are not part of the d.g.p in this segment, whereas maintain  $x_{2t}$ ,  $x_{3t}$  and  $x_{5t}$ .



## 2. SIMULATION EXERCISES (CONT.)

- We can see in **Figure 2** the PMs of the coefficients that are part of the d.g.p.
- **The PMs follow the true process.** This indicates that our methodology captures the dynamic of the d.g.p.
- In addition, we found that the PMs of the other variables have means approximately equal to zero.

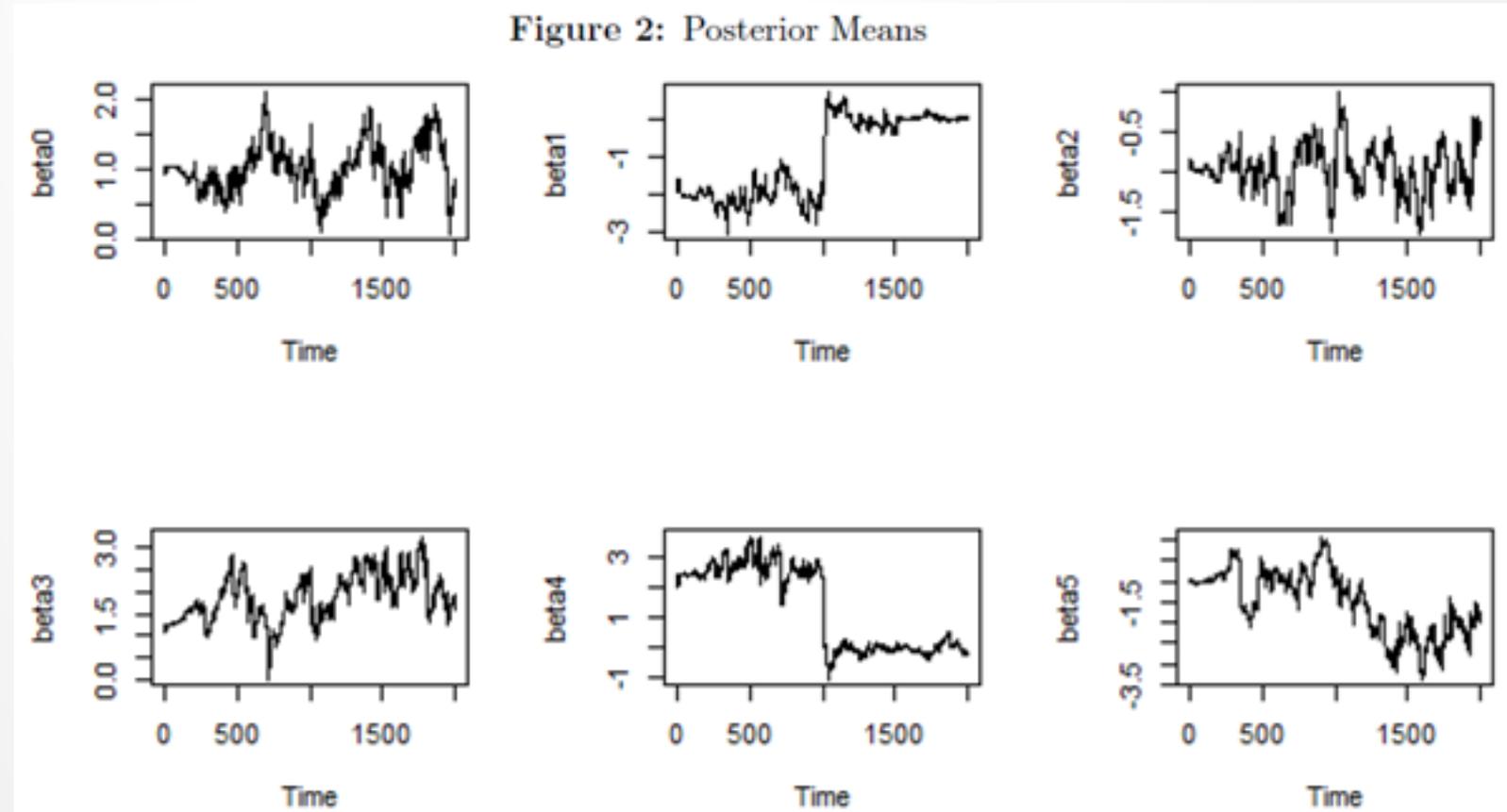


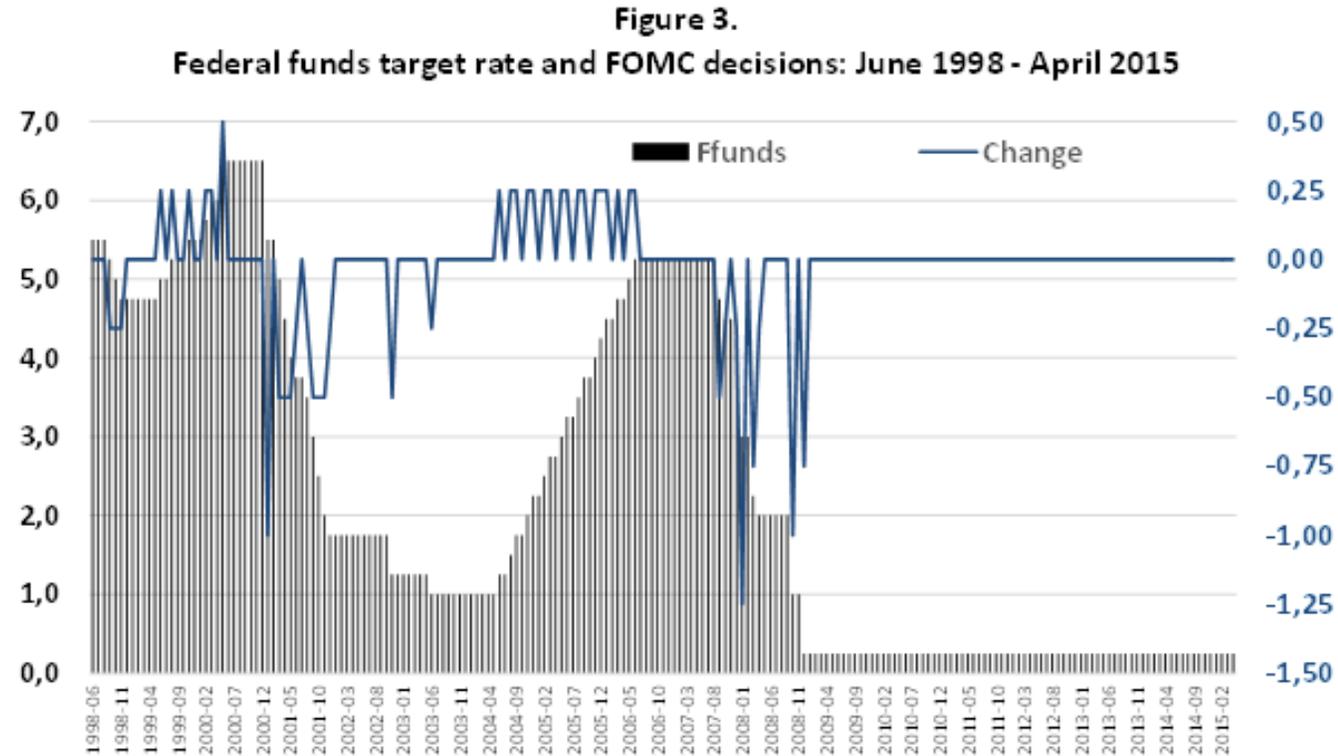
Table 1: Conditions under which data were generated

	$\beta_{0t}$	$\beta_{1t}$	$\beta_{2t}$	$\beta_{3t}$	$\beta_{4t}$	$\beta_{5t}$
$t = 1, \dots, 4000$	1	-2	-1	$1 + t/5000$	2.5	-1
$t = 4001, \dots, 5000$	1	0	-1	$1 + t/5000$	0	-2

The **d.g.p**  
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### 3. DATA & TARGET RATE CHARACTERISTICS

- We investigate the **federal funds target rate** at a monthly frequency (203) for the period from **June 1998 until April 2015**.
- This sample period covers:
  - Alan Greenspan's term** from Aug. 11, 1987 to Jan. 31, 2006, as well as
  - Ben S. Bernanke's term** from Feb. 1, 2006 to Jan. 31, 2014, and
  - Janet L. Yellen's term** from Feb. 3, 2014 until today.



**Table 4**

Summary statistics

FOMC decisions		Federal funds target rate	
# Decreases	23	Mean	2,30
# No-changes	157	Minimum (Dec 2008 - April 2015)	0,25
# Increases	23	Maximum (May - Dec. 2000)	6,50
		Standard deviation	2,15

Notes: the table presents summary statistics for the federal funds target rate and the FOMC decisions during the period June 1998 - April 2015 (203 months).



In **2008**, the Federal Reserve undertook **non-traditional monetary policy measures** to provide additional support to the economy.

### 3. DATA & TARGET RATE CHARACTERISTICS (CONT.)

**Macroeconomic and financial market variables** considered as potential predictors for the FOMC target rate decisions:

- i. **CPI, IP, as well as the expectations (Bloomberg surveys) of y/y GDP and IP.** These variables are most closely related to the monetary policy objectives of the Federal Reserve.
- ii. **The second group of variables consists of financial market data, where the estimation of FRAs is derived from both: the short-end of the OIS curve and US Treasury yield curve, specifically the 3m and 6m tenors (T-bills).**

**Table 5**

Set of potential predictors

Variable	Abbrev.	$Pr[\gamma_k = 1   \mathbf{y}]$ : PIPs	
		Dynamic BMA-MC3 Up	Down
<i>Panel A: Monetary policy variables</i>			
1. Inflation, CPI: U.S. city average: all items: seasonally adjusted	CPI.YoY	0,05	0,95
2. Industrial Production: seasonally adjusted	Ind.Pcc	1,00	1,00
3. Expected Industrial Production, survey of forecasters	EInd.Pcc	0,05	0,05
4. Expected year-over-year GDP, real GDP, survey of forecasters	EGDP.YoY	0,00	0,00
<i>Panel B: Financial and market variables</i>			
5. FRAs derived from the short-end of the OIS Swap curve (3x6)	FS3x6	0,95	0,00
6. FRAs derived from the short-end of US Treasurys curve, T-bills (3x6)	FUST3x6	1,00	0,05
7. Interest rate spread, 6-month T-bill less Fed funds	X6mFFunds	0,95	0,05

*Table 5* presents the set of candidate predictor variables in the BMA model for the FOMC decision on the federal funds target rate. The columns headed  $Pr[\gamma_k = 1 | \mathbf{y}]$  give the PIPs in the dynamic model for Up & Down movements in the fed funds rate on the full sample period June 1998 - April 2015.

# 4. EMPIRICAL RESULTS: ESTIMATION

- Based on the marginal PIPs  $Pr[\gamma_k = 1|y]$ , ( $k = 1, \dots, K$ ) a limited number of predictor variables are informative for the target rate decisions.
- For the **Logistic DMA-Up** model we find that **4** variables have conditional **PIPs > 0,50**, and **2** variables in the **Logistic DMA-Down**.

Table 6  
Properties of marginal posterior distributions

Logistic DMA - Up					
Parameter	Mean	St.D.	Percentiles		Parameter $Pr[\gamma_k = 1   y]$
			5th	95th	
CPI.YoY	0,006	0,011	0,000	0,024	0,050
→ Ind.Pcc	1,377	0,036	1,346	1,442	1,000
EInd.Pcc	0,016	0,027	0,001	0,063	0,050
EGDP.YoY	0,000	0,000	0,000	0,000	0,000
→ FS3x6	3,943	0,128	3,714	4,055	0,950
→ FUST3x6	-3,691	0,151	-3,845	-3,426	1,000
→ X6mFFunds	6,084	0,285	5,606	6,418	0,950

Logistic DMA - Down					
Parameter	Mean	St.D.	Percentiles		Parameter $Pr[\gamma_k = 1   y]$
			5th	95th	
→ CPI.YoY	0,114	0,074	-0,047	0,199	0,950
→ Ind.Pcc	-0,647	0,087	-0,840	-0,516	1,000
EInd.Pcc	-0,004	0,007	-0,021	0,000	0,050
EGDP.YoY	0,000	0,000	0,000	0,000	0,000
FS3x6	-0,007	0,022	-0,064	0,000	0,000
FUST3x6	0,001	0,023	-0,015	0,056	0,050
X6mFFunds	0,000	0,000	0,000	0,000	0,050

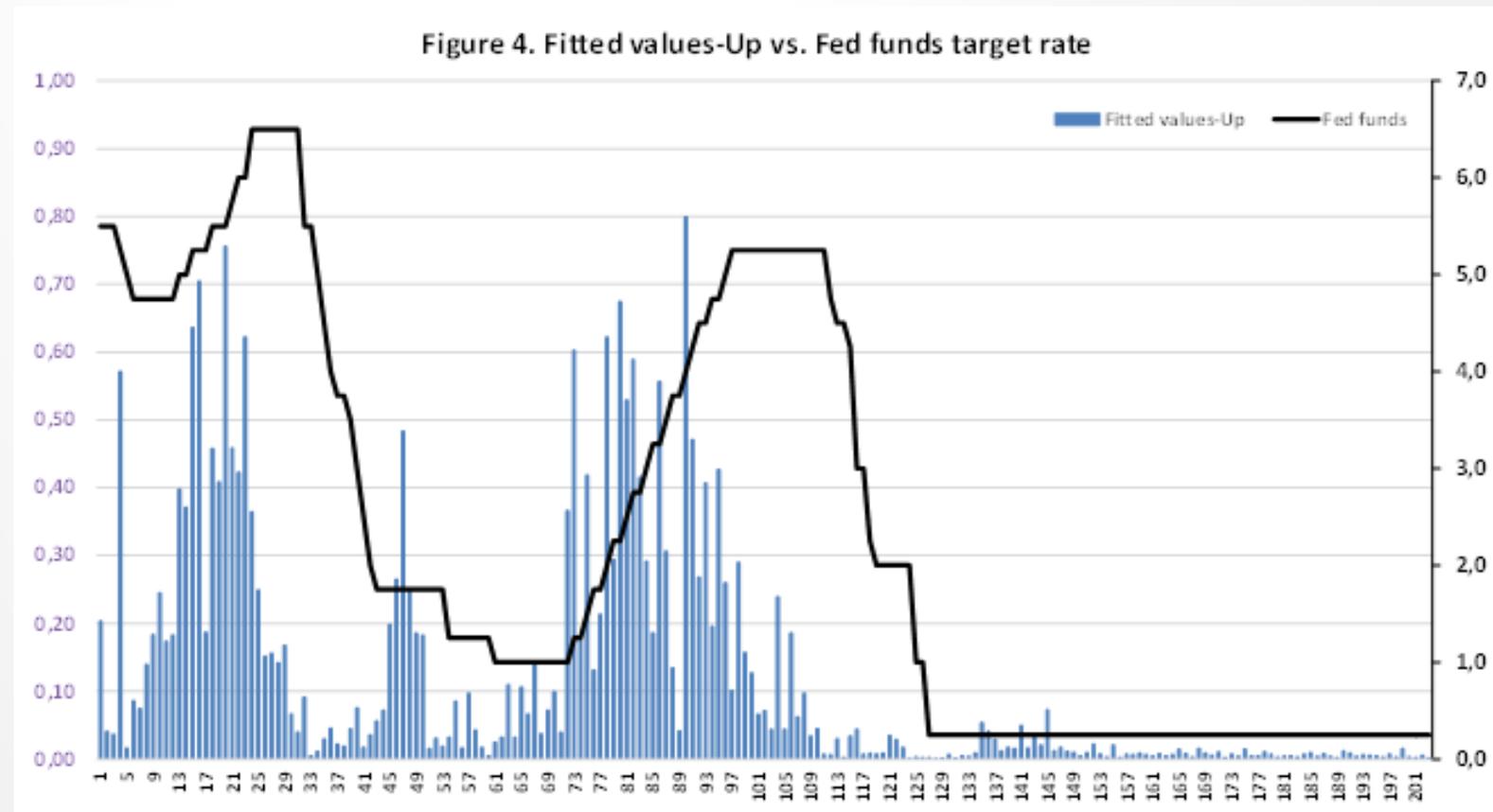
Dynamic Logit - Up				
Parameter	Mean	St.D.	Percentiles	
			5th	95th
CPI.YoY	1,597	0,020	1,574	1,646
Ind.Pcc	2,166	0,009	2,154	2,176
EInd.Pcc	0,936	0,032	0,918	0,998
EGDP.YoY	0,014	0,002	0,013	0,018
FS3x6	6,123	0,056	6,029	6,196
FUST3x6	-6,127	0,040	-6,186	-6,046
X6mFFunds	5,455	0,186	4,992	5,591

Dynamic Logit - Down				
Parameter	Mean	St.D.	Percentiles	
			5th	95th
CPI.YoY	0,252	0,019	0,224	0,279
Ind.Pcc	-0,073	0,034	-0,109	0,004
EInd.Pcc	-0,852	0,095	-0,973	-0,688
EGDP.YoY	-0,160	0,007	-0,171	-0,150
FS3x6	-3,261	0,049	-3,348	-3,197
FUST3x6	3,642	0,036	3,561	3,691
X6mFFunds	-1,653	0,032	-1,689	-1,603

We find that market expectations embedded in IRDs such as **FRAs** derived from the **OIS** and the **U.S. Treasury yield curve**, as well as expectations from the spread **X6mFFunds**, represent short-term market expectations about inflation and economic activity to which the FOMC does react.

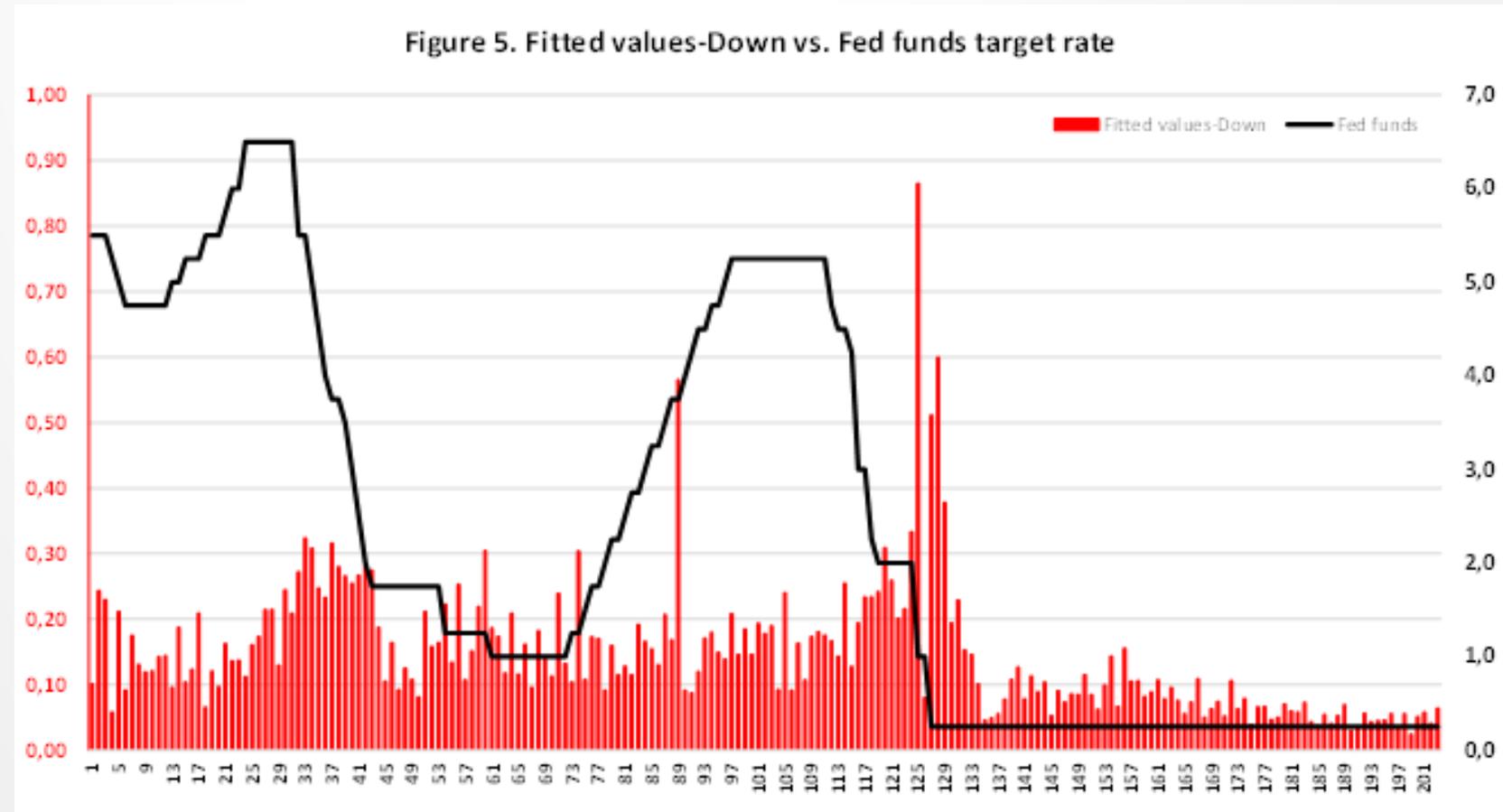
## 4. EMPIRICAL RESULTS: FORECASTING

- Probabilities of: (i) **increase**, (ii) **no-change** or (iii) **decrease** of the target rate for each month in the full sample period June 1998-April 2015.
- We compared the **fitted values** vs. the realized FOMC's decisions.
- **Fig. 4** shows that *Up* movements in the target repo rate are very well anticipated by the **Logistic DMA-Up model**, with probabilities ranging from **0,40** up to **0,80**.



## 4. EMPIRICAL RESULTS: FORECASTING (CONT.)

- The case for **Down movements** is quite different, because only from August 2007 to November 2008 the **Logistic DMA** shows **high levels of probabilities** in the range of **0,25** to **0,865**.



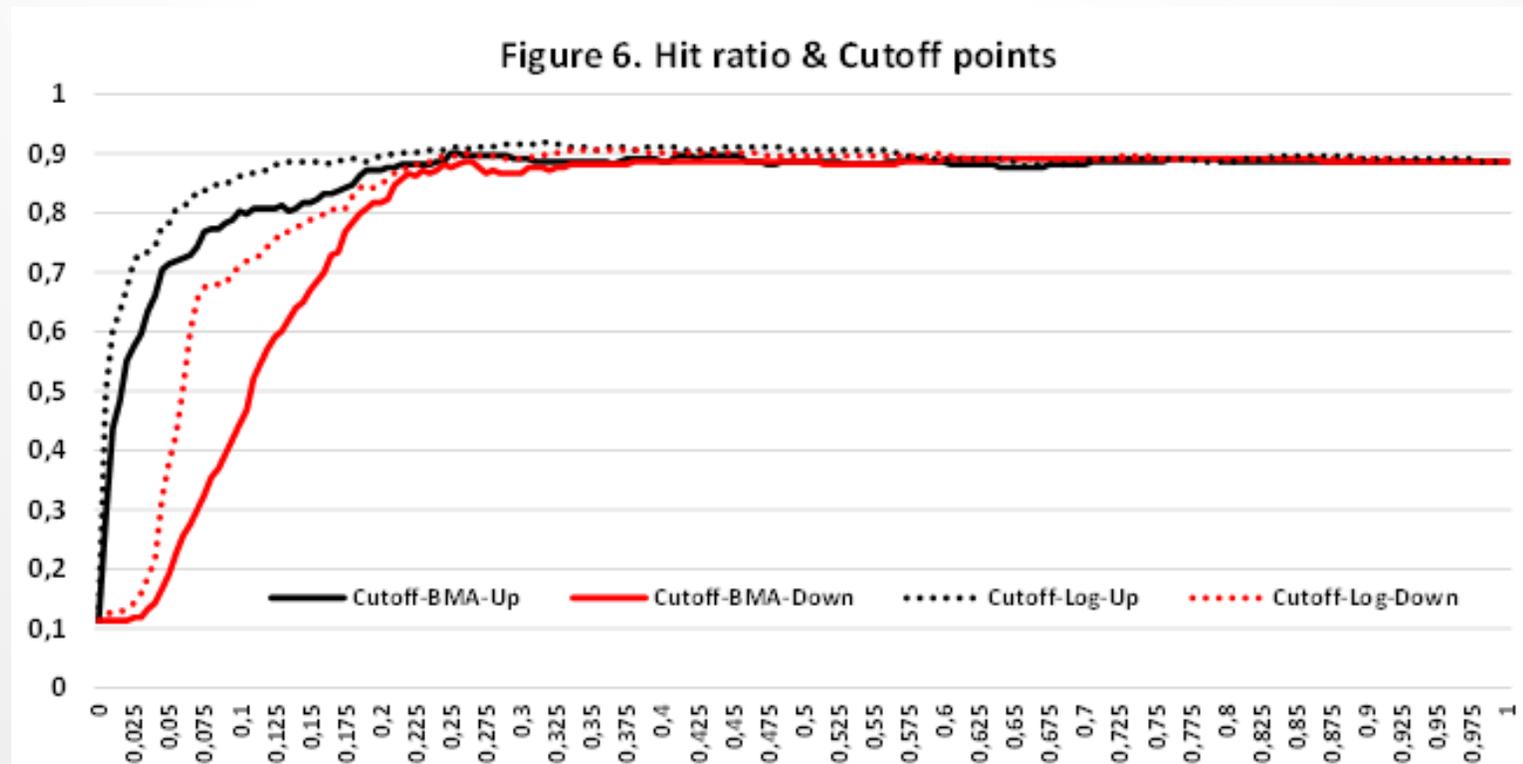
## 4. EMPIRICAL RESULTS: FORECASTING (CONT.)

- Hit rates are equal to the percentage of correctly predicted target rate decisions.
- In-sample refers to hit rates for probability estimates obtained when the models are estimated on the full sample period June 1998-April 2015.
- ROC Curve: The **Logistic DMA-Up** and **Dynamic Logit-Up** models present high hit ratios of **87,2** and **88,7**, respectively. Meanwhile for the **Logistic DMA-Down** and **Dynamic Logit-Down** models are **79,8** and **68,0**, respectively.

Table 7

Hit rates and cutoff points

Model	Data	In-sample hit rate	Cutoff point
Logistic DMA-Up	Real time	87,2 (177 / 203)	0,186
Logistic DMA-Down	Real time	79,8 (162 / 203)	0,186
Dynamic Logit-Up	Real time	88,7 (180 / 203)	0,164
Dynamic Logit-Down	Real time	68,0 (138 / 203)	0,073



## 4. EMPIRICAL RESULTS: FORECASTING (CONT.)

- As Youden (1950) and Sebastian et al (2010) describe, the **cutpoint analysis** presented in last column of Table 7 involves locating the **optimal value that minimizes prediction errors associated with binary outcomes**, where both, the sensitivity and specificity statistical measures of the performance of the binary classification are maximized.
- **Sensitivity\*** (also called the true positive rate-TP) **measures the proportion of positives which are correctly identified as such** (e.g., the model assigns a high probability of occurrence to the event in which the FOMC decides to increase/decrease the target repo rate, and the final outcome is True). This measure is complementary to the false negative rate-FN.
- On the other hand, **specificity\*** (also called the true negative rate-TN) **measures the proportion of negatives which are correctly identified as such** (e.g., the model assigns a low probability of occurrence to the event in which the FOMC decides to increase/decrease the target repo rate, and the final outcome is True), and is complementary to the false positive rate-FP.

*\* Both measures can be represented graphically as a **receiver operating characteristic curve** or **ROC curve**. **Fig. 6** portrays the corresponding ROC curves for each model considered in this analysis.*

## 5. CONCLUSIONS

- An important contribution of this paper is the **BMA scheme with dynamic betas**, that takes into account model uncertainty by going through all combinations of models that can arise within a given set of variables on a real-time basis to construct in-sample probability forecasts.
- In addition, to estimate our models, **we extend the DMA procedure for dynamic logistic regressions** developed by [McCormick et al. \(2012\)](#). In particular, we implement a **MC<sup>3</sup> procedure for model selection**, reducing enormously the computational burden of the algorithm.
- **FOMC meetings during the sample period June 1998-April 2015 are predicted very well:** Logistic DMA-Up and Dynamic Logit-Up models present high **hit ratios of 87,2 and 88,7**. Meanwhile, the Logistic DMA-Down and Dynamic Logit-Down models have medium-high **hit ratios: 79,8 and 68,0**, respectively.
- Our empirical results show strong evidence for persistence in the target rate decisions. For the **Logistic DMA-Up model**, the most predictive ability is found for, **first, economic activity measures like IP**, and **second, term structure variables such as 3x6 FRAs and IR spreads**. For the **Logistic DMA-Down**, the most predictive ability rests on the following **macroeconomic variables: IP and y/y CPI**. In this case, term structure variables do not present evidence of predictive signals.

## 6. FUTURE RESEARCH ENDEAVORS

The following ideas could be explored in order to improve the results and the methodology presented in this paper:

- i. **Design and algorithm that takes into account the trinomial case** (multinomial classification), where the three (3) possible states of the FOMC decisions could be very well captured.
- ii. **Perform ‘*pattern net*’ analysis associated with neural networks classification** and compare the results with the output provided by our model: the Bayes classification approach.
- iii. **Conduct this analysis for the Economic and Monetary Union (EMU)** and contrast the results by running out-of-sample estimates in order to stress the model and test its predictive power throughout the sample period.
- iv. **...among others.**

**Thank you!**