LOG-SKEW-NORMAL MIXTURE MODEL FOR OPTION VALUATION

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Agenda

INTRODUCTION

- There is empirical evidence that financial stock returns are not normally distributed but are characterized by skewness, leptokurticity, heavy-tailedness and other non-Gaussian properties.
- The skewness and kurtosis of the empirical distribution function (EDF) of stock returns contribute significantly to the phenomenon of volatility smile.
- In recent years, there have been considerable efforts to report that the unconditional probability distributions of returns on financial stocks are not normally distributed.
- Specifically, these distributions tend to have heavier tails (leptokurtic) and asymmetry.

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- The Skew Brownian motion has been used for pricing European options ([Corns and Satchell(2007)]).
- The Log-Skew-Normal (LSN) distribution, which is an extension for the positive data of the Log-Normal (LN) distribution used to form data with asymmetry and kurtosis.

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Objective

In this talk

- By assuming that the stock distribution follows a Log-Skew-Normal mixture (LSNMIX) distribution, we calculate an explicit formula for option valuation for both European call and put options and Greek measures.
- We also show that some of the well-known models are obtained as special cases from the proposed model([Black and Scholes(1973)], [Bahra(1997)] and [Corns and Satchell(2007)]).
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LOG-SKEW-NORMAL DISTRIBUTION (LSN)

- The LSN distribution was first introduced by [Azzalini et al.(2003)Azzalini, Dal Cappello, and Kotz] and later by [Lin and Stoyanov(2009)].
- A random variable Y has a LSN distribution with asymmetry parameter $\lambda \in \mathbb{R}$, denoted as $Y \sim LSN\left(\Lambda_1\right)$, if its *pdf* is of the form

$$
\begin{aligned}
f_Y\left(y;\Lambda_1\right) =& \frac{2}{\sigma y}\varphi\left(\frac{\ln y-\mu}{\sigma}\right)\Phi\left(\lambda\frac{\ln y-\mu}{\sigma}\right) \\
=& \frac{1}{y}\phi_{SN}\left(\ln y;\Lambda_1\right), \qquad y \in \mathbb{R}^+,
\end{aligned} \tag{1}
$$

where $\Lambda_1 = \left(\mu,\sigma,\lambda\right)$, $\sigma > 0$, $\phi_{SN}(\cdot)$ denotes the *pdf* of the skew-Normal (SN) distribution, and $\varphi\left(\cdot\right)$ and $\Phi(\cdot)$ denote the *pdf* and *cdf* of a standard univariate normal variable, respectively.

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LOG-SKEW-NORMAL DISTRIBUTION

If $\mu = 0$ and $\sigma = 1$ then Y is said to have a (standard) LSN distribution, i.e., $Y \sim LSN(\lambda)$. The parameter λ controls the skewness, which is positive when $\lambda > 0$ and negative when $\lambda < 0$. The *cdf* of (1) is given by

$$F_Y(y; \Lambda_1) = \Phi\left(\frac{\ln y - \mu}{\sigma}\right) - 2T\left(\frac{\ln y - \mu}{\sigma}; 0, \lambda\right), \quad (2)$$

where the function $T(z; \alpha, \lambda)$ with $\alpha \geq 0$ is given as

$$T(z; \alpha, \lambda) = sign(\lambda)\left[\frac{\arctan(|\lambda|)}{2\pi} - \int_\alpha^z \int_0^{|\lambda|x} \varphi(x, \alpha, 1)\varphi(y) \mathrm{d}y\mathrm{d}x\right],$$

and $sign(\cdot)$ is the signum function.

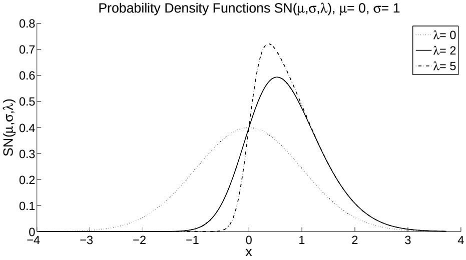

FIGURE: Comparison of the SN *pdf* with $\lambda > 0$.

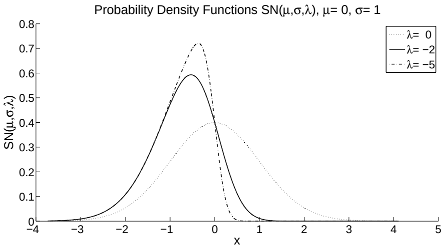

FIGURE: Comparison of the SN *pdf* with $\lambda < 0$.

Skew Normal Mixtures

we consider the finite mixture model suggested by [Lin et al.(2007)Lin, Lee, and Yen]: We assume that if Y is SNMIX distributed then the transformation $X = \exp\{Y\}$ is distributed as a LSNMIX. Let us assume that $f_Y(y)$ is the weighted sum of m-component SNMIX densities, that is,

$$f_Y(y;\Lambda) = \sum_{j=1}^{m} \omega_j \phi_{SN}(y;\mu_j,\sigma_j,\lambda_j). \quad (3)$$

We use the notation $Y \sim SNMIX(\Lambda)$, where $\Lambda = (\xi_{\mathbf{1}},\ldots,\xi_{\mathbf{m}})$, and $\xi_{\mathbf{j}} = (\omega_j,\mu_j,\sigma_j,\lambda_j)$ is the parameter vector that defines the j-th component and probability weights, ω_j, satisfy the conditions

$$\sum_{j=1}^{m} \omega_j = 1, \qquad 0 < \omega_j < 1, \text{ for each } j. \quad (4)$$

The choice of a finite mixture is attractive from the application view point because of its flexibility and allows us to consider different kinds of shaped distributions. For instance, the two component SNMIX model has the advantage of numerical tractability, because it has only seven parameters. Assuming $\xi_{\mathbf{j}} = (\omega_j, \mu_j, \sigma_j, \lambda_j)$, with $\mu_1 = -1$, $\mu_2 = 1$, $\sigma_1 = \sigma_2 = 1$, $\lambda_1 = .5$ and $\lambda_2 = -2$. Figure 3 shows the *pdf* shape of the SNMIX for three values of ω.

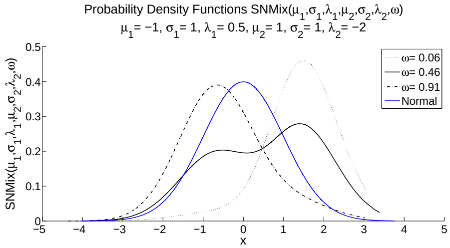

FIGURE: Comparison of the *pdf* of the SNMIX with varying ω.

Option pricing

Let S_t be the price of the underlying stock at time t and $C(t,\tau;K)$ the price of the call option with strike price K and maturity date of $T=t+\tau$. It is assumed that r is the annual risk-free rate.
According to [Harrison and Pliska(1981)], in the absence of arbitration, the price of European call options can be written as follows:

$$C(t,\tau;K) = \mathbb{E}\left[e^{-r\tau}\max\{S_T-K,0\}\right] = e^{-r\tau}\mathbb{E}\left[\max\{S_T-K,0\}\right]$$

$$C_t(K) \quad = e^{-r\tau}\int_K^{\infty}(S_T-K)f(S_T)\,\mathrm{d}S_T.$$

Here $\mathbb{E}[.]$ is the expected value conditional (risk neutral) on any information that is available at time t,
$f(S_T)$ is the risk-neutral pdf (risk-neutral distribution, *RND*) for the underlying stock.
In an arbitrage-free economy, the stock price discounted by the risk free rate becomes martingale, that is:

$$\mathbb{E}[e^{-r\tau}S_T] = S_t, \qquad \tau > 0 \qquad (5)$$

Option price using the LSNMIX

We define the distribution of the logarithm of the stock price S_T using its location and scale parameters A and B, respectively, and also Λ, the parameter of the SNMIX. These parameters satisfy the following relationship:

$$\ln\left[S_T\right] = A + BY, \qquad \text{with} \qquad Y \sim SNMIX\left(\Lambda\right). \tag{6}$$

Then, the *pdf* of S_T is a LSNMIX.

Option price using the LSNMIX

Proposition

The price of a European call option is given by

$$e^{r\tau}\, C_t\left(K;\Lambda\right) = \sum_{j=1}^{m} \frac{2\omega_j \mathbb{E}\left[S_T\right]}{\Upsilon_j\left(\Lambda, B\right)} \int_{-\delta_{1j}}^{\infty} \varphi\left(z\right) \Phi\left[\lambda_j \widetilde{z}_j\right] \mathrm{d}z - \sum_{j=1}^{m} \omega_j K \left[1 - F_Y\left(-\delta_{2j};\lambda_j\right)\right], \tag{7}$$

where $\widetilde{z}_j = z + B\sigma_j$ *and* $F_Y(\cdot)$ *is given in* (2),

$$\delta_{1j} = \delta_{2j} + B\sigma_j, \qquad \delta_{2j} = -\frac{\kappa - \mu_j}{\sigma_j}, \qquad \kappa = \frac{\ln K - A}{B}, \tag{8}$$

$$\delta_{2j} = \frac{1}{B\sigma_j}\left\{\ln\left(\frac{\mathbb{E}\left[S_T\right]}{K}\right) - \ln\left[\Upsilon_j\left(\Lambda, B\right)\right]\right\} - \frac{1}{2}B\sigma_j,$$

Option price using the LSNMIX

Proposition

The LSNMIX European put option price is given by

$$e^{r\tau}P_t(K;\Lambda) = \sum_{j=1}^{m}\omega_j K F_Y\left(-\delta_{2j};\lambda_j\right) - \sum_{j=1}^{m}\frac{2\mathbb{E}\left[S_T\right]\omega_j}{\Upsilon_j\left(\Lambda,B\right)}\int_{-\infty}^{-\delta_{1j}}\varphi\left(z\right)\Phi\left[\lambda_j\widetilde{z}_j\right]\mathrm{d}z, \tag{10}$$

where δ_{1j} and δ_{2j} are given in (8) and $\Upsilon_j\left(\Lambda,B\right)$ is given in (9).

Using the option valuation formula, we can obtain the put-call parity relationship by subtracting expression (7) from (10) to obtain the following equality

$$e^{r\tau}\left(C_t(K;\Lambda) - P_t(K;\Lambda)\right) = \mathbb{E}\left[X_T\right] - K. \tag{11}$$

BLACK-SCHOLES

- Assuming $\xi_{\mathbf{j}} = \left(\frac{1}{m}, \mu, \sigma, 0\right)$ for all j in (3), substituting in expressions (7) and (10) yields, respectively,

$$
\begin{aligned}
e^{r\tau} C_t\left(K;\Lambda\right) =& \mathbb{E}\left[S_T\right] \Phi \left[\frac{1}{B\sigma} \ln\left[\frac{\mathbb{E}\left[S_T\right]}{K}\right] + \frac{1}{2} B\sigma\right] - K\Phi \left[\frac{1}{B\sigma} \ln\left[\frac{\mathbb{E}\left[S_T\right]}{K}\right] - \frac{1}{2}\right. \\
=& \mathbb{E}\left[S_T\right] \Phi\left(d_1\right) - K\Phi\left(d_2\right), \qquad (12)
\end{aligned}
$$

where

$$
d_2 = \frac{1}{B\sigma} \ln\left(\frac{\mathbb{E}\left[S_T\right]}{K}\right) - \frac{1}{2} B\sigma, \qquad (13)
$$

and $d_1 = d_2 + B\sigma$. Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula of [Black and Scholes(1973)].

[Bahra(1997)]

When $\xi_{\mathbf{j}} = \left(\omega_j, \mu_j^*, \sigma_j, 0\right)$ for all j, where $\mu_j^* = B\left(\mu_j - \frac{1}{2}\sigma_j^2\right)$ in (3), substituting in expressions (7) and (10) yields, respectively,

$$e^{r\tau}\, C_t\left(K; \Lambda\right) = \sum_{j=1}^{m} \frac{\omega_j \mathbb{E}\left[S_T\right]}{\Upsilon_j\left(\Lambda, B\right)} \Phi\left(\delta_{1j}\right) - K \sum_{j=1}^{m} \omega_j \Phi\left(\delta_{2j}\right), \tag{14}$$

where δ_{1j} and δ_{2j} are given in (8) and

$$\Upsilon_j\left(\Lambda, B\right) = \sum_{l=1}^{m} \omega_l \exp\left\{B^2\left(\mu_l - \mu_j\right)\right\}. \tag{15}$$

Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula given in [Bahra(1997)].

[Corns and Satchell(2007)]

When $\xi_{\mathbf{j}} = \left(\frac{1}{m}, \mu, \sigma, \lambda\right)$ for all j in (3), substituting in expressions (7) and (10) yields, respectively,

$$e^{r\tau} C_t\left(K;\Lambda\right) = \frac{\mathbb{E}\left[S_T\right]}{\Phi\left(\rho\sigma B\right)} \int_{-\delta_1}^{\infty} \varphi\left(z\right) \Phi\left[\lambda\left(z + B\sigma\right)\right] \mathrm{d}z - K\left[1 - F_Y\left(-\delta_2;\lambda\right)\right], \tag{16}$$

where

$$\delta_2 = \frac{1}{B\sigma} \ln\left\{\frac{\mathbb{E}\left[S_T\right]}{2K\,\Phi\left(\rho\sigma B\right)}\right\} - \frac{1}{2}B\sigma, \tag{17}$$

and $\delta_1 = \delta_2 + B\sigma$. Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula in [Corns and Satchell(2007)].

Greek Measure-Delta

The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying stock.

$$\Delta_{\text{Call}} = \sum_{j=1}^{m} \frac{2\omega_j}{\Upsilon_j(\Lambda, B)} \int_{-\delta_{1j}}^{\infty} \varphi(z)\, \Phi\left[\lambda_j \widetilde{z}_j\right] \mathrm{d}z,$$

$$\Delta_{\text{Put}} = \Delta_{\text{Call}} - 1.$$

Greek Measure-Gamma

The gamma of a portfolio of options on an underlying stock is the rate of change of the portfolio's delta with respect to the price of the underlying stock, i.e., the second partial derivative of the portfolio with respect to stock price

$$\Gamma_{\text{Call}} = \Gamma_{\text{Put}} = \sum_{j=1}^{m} 2\omega_j \frac{\Phi\left[-\lambda_j \delta_{2j}\right]}{\Upsilon_j\left(\Lambda, B\right)} \frac{\varphi\left(\delta_{1j}\right)}{B\sigma_j S}. \quad (18)$$

Greek Measures

OPTION PRICING BASED ON A LOG-SKEW-NORMAL MIXTURE
by
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- We now present an example from real market data to model the distribution of the stock prices and compare the numerical values of a European option under the assumption that the stock movement follow a LSNMIX distribution.
- The market data of the $S\&P500$ index for the daily prices were considered from January 4, 2010 to October 13, 2014.

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Statistics	Values
Mean	0.0004
Stan. Dev.	0.0102
Minimum	-0.0690
Maximum	0.0463
Skewness	-0.4854
Kurtosis	7.6325
JB test	1120.1201

TABLE: Summary of the Descriptive Statistics

The empirical distribution and its descriptive statistics presented in table 1 are analysed using the test proposed by Jarque & Bera (1987), in particular, the skewness and kurtosis, which confirms that the null hypothesis of a normal distribution must be rejected.

Estimate	MME	MLE
μ_1	-0.0039	-0.0025
μ_2	0.0036	0.0045
σ_1	0.0056	0.0056
σ_2	0.0137	0.0132
λ_1	1.4683	1.4686
λ_2	-0.6444	-0.6457
ω	0.6158	0.6158

TABLE: Estimates for adjusting the $SNMIX\,(\Lambda)$

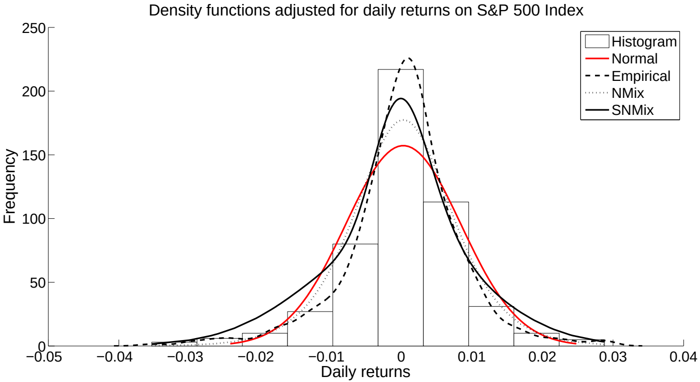


FIGURE: Returns vs. normal distribution and SNMIX.

Strike (K)	Maturity			
	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 1.0$
1300	122,7236	146,2137	168,1804	188,9434
1350	80,8813	106,9346	130,0796	151,5625
1400	47,3226	73,8094	96,9744	118,4221
1450	24,1159	47,8415	69,5369	90,0320
1500	10,5793	29,0350	47,9058	66,5672

TABLE: Comparison prices of call option Black Scholes.

Strike (K)	Maturity			
	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 1.0$
1300	123,0441	143,9754	164,5054	184,3572
1350	78,9742	103,5501	125,8430	146,8139
1400	44,7446	70,3303	92,9945	114,1851
1450	22,2624	45,2095	66,5457	86,9097
1500	10,7043	28,0958	46,5603	65,1000

TABLE: Comparison prices of call option Corrado & Su.

Strike (K)	Maturities			
	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 1.0$
1300	121,8372	143,5546	164,1304	183,8455
1350	76,7124	99,9953	121,4675	141,8595
1400	35,8423	59,5374	81,2789	101,9446
1450	8,3470	25,7672	45,2634	65,0156
1500	1,6477	8,0504	19,5020	34,6045

TABLE: Comparison prices of call option SNMIX.

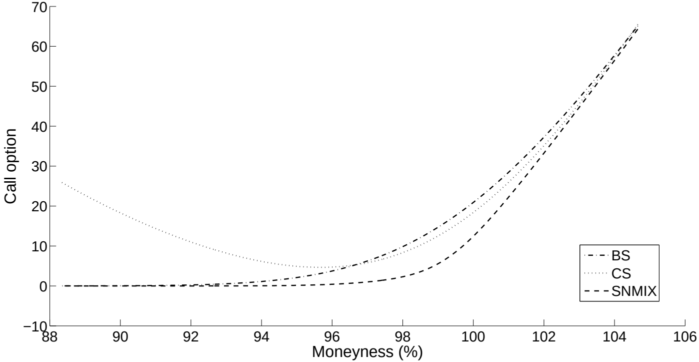
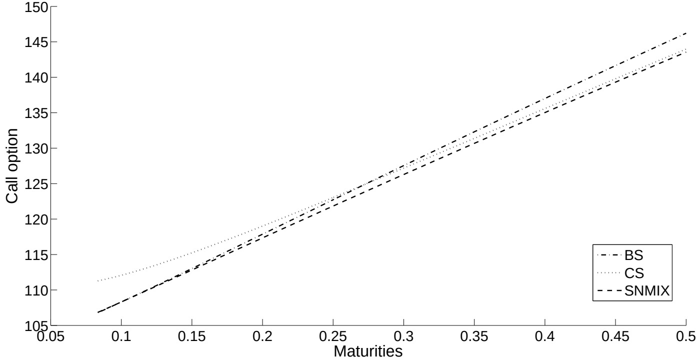


FIGURE: Call option for different strikes.



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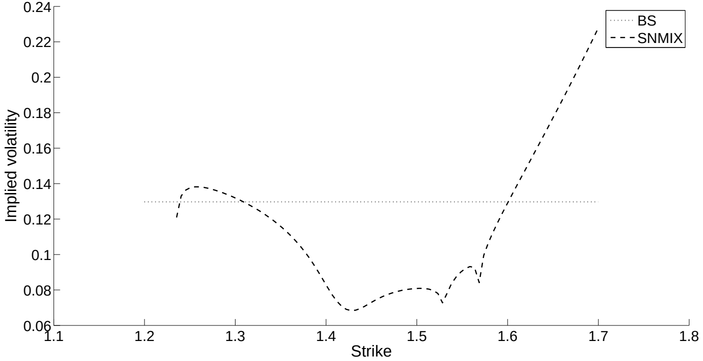

FIGURE: Implied volatility.

Conclusions

- We have proposed an alternative approach for calculating the price of the call and put options when the stock return distribution follows the SNMIX distribution.
- We have obtained an explicit expression for the price of the European options, and from the proposed model, we deduce some well-known models, such as [Black and Scholes(1973)], [Bahra(1997)] and [Corns and Satchell(2007)].
- An example from real market data is also presented to implement the proposed model, and comparisons are made with well-known models.
- The derivation of an expression for the implied volatilities at and around-the money and studies of its asymptotic behaviour are in progress.

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THANK YOU VERY MUCH

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