Log-Skew-Normal mixture model for option valuation

J.A. Jiménez and V. Arunachalam

Department of Statistics
Universidad Nacional de Colombia
Bogotá, Colombia
josajimenezm@unal.edu.co varunachalam@unal.edu.co

ICASQF 2016 Cartagena
There is empirical evidence that financial stock returns are not normally distributed but are characterized by skewness, leptokurticity, heavy-tailedness and other non-Gaussian properties.

The skewness and kurtosis of the empirical distribution function (EDF) of stock returns contribute significantly to the phenomenon of volatility smile.

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Specifically, these distributions tend to have heavier tails (leptokurtic) and asymmetry.
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- By assuming that the stock distribution follows a Log-Skew-Normal mixture (LSNMIX) distribution, we calculate an explicit formula for option valuation for both European call and put options and Greek measures.

- We also show that some of the well-known models are obtained as special cases from the proposed model ([Black and Scholes(1973)], [Bahra(1997)] and [Cornes and Satchell(2007)]).

- An example from S&P500 daily returns to price is presented to illustrate the model.
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The LSN distribution was first introduced by [Azzalini et al. (2003) Azzalini, Dal Cappello, and Kotz] and later by [Lin and Stoyanov (2009)].

A random variable $Y$ has a LSN distribution with asymmetry parameter $\lambda \in \mathbb{R}$, denoted as $Y \sim LSN (\Lambda_1)$, if its pdf is of the form

$$f_Y(y; \Lambda_1) = \frac{2}{\sigma y} \varphi \left( \frac{\ln y - \mu}{\sigma} \right) \Phi \left( \lambda \frac{\ln y - \mu}{\sigma} \right)$$

$$= \frac{1}{y} \phi_{SN} (\ln y; \Lambda_1), \quad y \in \mathbb{R}^+,$$

where $\Lambda_1 = (\mu, \sigma, \lambda)$, $\sigma > 0$, $\phi_{SN}(\cdot)$ denotes the pdf of the skew-Normal (SN) distribution, and $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the pdf and cdf of a standard univariate normal variable, respectively.
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If $\mu = 0$ and $\sigma = 1$ then $Y$ is said to have a (standard) LSN distribution, i.e., $Y \sim LSN(\lambda)$. The parameter $\lambda$ controls the skewness, which is positive when $\lambda > 0$ and negative when $\lambda < 0$. The cdf of (1) is given by

$$F_Y(y; \Lambda_1) = \Phi \left( \frac{\ln y - \mu}{\sigma} \right) - 2T \left( \frac{\ln y - \mu}{\sigma} ; 0, \lambda \right), \quad (2)$$

where the function $T(z; \alpha, \lambda)$ with $\alpha \geq 0$ is given as

$$T(z; \alpha, \lambda) = \text{sign} (\lambda) \left[ \frac{\arctan(|\lambda|)}{2\pi} - \int_{\alpha}^{z} \int_{0}^{\left|\lambda\right|x} \varphi(x, \alpha, 1) \varphi(y) dy \, dx \right],$$

and $\text{sign}(\cdot)$ is the signum function.
**Figure:** Comparison of the SN pdf with $\lambda > 0$. 
Figure: Comparison of the SN pdf with $\lambda < 0$. 
we consider the finite mixture model suggested by [Lin et al.(2007)Lin, Lee, and Yen]: We assume that if $Y$ is SNMIX distributed then the transformation $X = \exp \{Y\}$ is distributed as a LSNMIX. Let us assume that $f_Y(y)$ is the weighted sum of $m$-component SNMIX densities, that is,

$$f_Y(y; \Lambda) = \sum_{j=1}^{m} \omega_j \phi_{SN}(y; \mu_j, \sigma_j, \lambda_j).$$ (3)

We use the notation $Y \sim \text{SNMIX} (\Lambda)$, where $\Lambda = (\xi_1, \ldots, \xi_m)$, and $\xi_j = (\omega_j, \mu_j, \sigma_j, \lambda_j)$ is the parameter vector that defines the $j$-th component and probability weights, $\omega_j$, satisfy the conditions

$$\sum_{j=1}^{m} \omega_j = 1, \quad 0 < \omega_j < 1, \text{ for each } j.$$ (4)
The choice of a finite mixture is attractive from the application viewpoint because of its flexibility and allows us to consider different kinds of shaped distributions. For instance, the two component SNMIX model has the advantage of numerical tractability, because it has only seven parameters. Assuming \( \xi_j = (\omega_j, \mu_j, \sigma_j, \lambda_j) \), with \( \mu_1 = -1, \mu_2 = 1, \sigma_1 = \sigma_2 = 1, \lambda_1 = .5 \) and \( \lambda_2 = -2 \). Figure 3 shows the pdf shape of the SNMIX for three values of \( \omega \).
The Skew Normal Mixtures

Probability Density Functions SNMix(\(\mu_1, \sigma_1, \lambda_1, \mu_2, \sigma_2, \lambda_2, \omega\))

\(\mu_1 = -1, \sigma_1 = 1, \lambda_1 = 0.5, \mu_2 = 1, \sigma_2 = 1, \lambda_2 = -2\)

\(\omega = 0.06\)
\(\omega = 0.46\)
\(\omega = 0.91\)

Normal

**Figure:** Comparison of the *pdf* of the SNMIX with varying \(\omega\).
Let $S_t$ be the price of the underlying stock at time $t$ and $C(t, \tau; K)$ the price of the call option with strike price $K$ and maturity date of $T = t + \tau$. It is assumed that $r$ is the annual risk-free rate.

According to [Harrison and Pliska(1981)], in the absence of arbitrage, the price of European call options can be written as follows:

$$C(t, \tau; K) = \mathbb{E}\left[e^{-r\tau} \max\{S_T - K, 0\}\right] = e^{-r\tau} \mathbb{E}\left[\max\{S_T - K, 0\}\right]$$

$$C_t(K) = e^{-r\tau} \int_{K}^{\infty} (S_T - K) f(S_T) dS_T.$$ 

Here $\mathbb{E}[]$ is the expected value conditional (risk neutral) on any information that is available at time $t$, $f(S_T)$ is the risk-neutral pdf (risk-neutral distribution, $RND$) for the underlying stock.

In an arbitrage-free economy, the stock price discounted by the risk free rate becomes martingale, that is:

$$\mathbb{E}[e^{-r\tau} S_T] = S_t,$$ 

$$\tau > 0$$ (5)
We define the distribution of the logarithm of the stock price $S_T$ using its location and scale parameters $A$ and $B$, respectively, and also $\Lambda$, the parameter of the SNMIX. These parameters satisfy the following relationship:

$$\ln[S_T] = A + BY, \quad \text{with} \quad Y \sim \text{SNMIX} (\Lambda). \quad (6)$$

Then, the pdf of $S_T$ is a LSNMIX.
The price of a European call option is given by

\[ e^{r\tau} C_t(K; \Lambda) = \sum_{j=1}^{m} \frac{2\omega_j \mathbb{E}[S_T]}{\Upsilon_j(\Lambda, B)} \int_{-\delta_{1j}}^{\infty} \varphi(z) \Phi[\lambda_j \tilde{z}_j] dz \]

\[- \sum_{j=1}^{m} \omega_j K \left[ 1 - F_Y(-\delta_{2j}; \lambda_j) \right], \tag{7} \]

where \( \tilde{z}_j = z + B\sigma_j \) and \( F_Y(\cdot) \) is given in (2),

\[ \delta_{1j} = \delta_{2j} + B\sigma_j, \quad \delta_{2j} = -\frac{\kappa - \mu_j}{\sigma_j}, \quad \kappa = \frac{\ln K - A}{B}, \tag{8} \]

\[ \delta_{2j} = \frac{1}{B\sigma_j} \left\{ \ln \left( \frac{\mathbb{E}[S_T]}{K} \right) - \ln \left[ \Upsilon_j(\Lambda, B) \right] \right\} - \frac{1}{2} B\sigma_j, \]
**Option price using the LSNMIX**

**Proposition**

The LSNMIX European put option price is given by

\[
e^{r\tau} P_t(K; \Lambda) = \sum_{j=1}^{m} \omega_j K F_Y(-\delta_2 j; \lambda_j) - \sum_{j=1}^{m} \frac{2 \mathbb{E} [S_T] \omega_j}{\Upsilon_j(\Lambda, B)} \int_{-\infty}^{-\delta_1 j} \varphi(z) \Phi[\lambda_j \tilde{z}_j] dz,
\]

where \(\delta_1 j \) and \(\delta_2 j \) are given in (8) and \(\Upsilon_j(\Lambda, B) \) is given in (9).

Using the option valuation formula, we can obtain the put-call parity relationship by subtracting expression (7) from (10) to obtain the following equality

\[
e^{r\tau} (C_t(K; \Lambda) - P_t(K; \Lambda)) = \mathbb{E} [X_T] - K.
\]
Assuming $\xi_j = \left( \frac{1}{m}, \mu, \sigma, 0 \right)$ for all $j$ in (3), substituting in expressions (7) and (10) yields, respectively,

$$
e^{r\tau} C_t(K; \Lambda) = \mathbb{E} \left[ S_T \right] \Phi \left[ \frac{1}{B\sigma} \ln \left( \frac{\mathbb{E} \left[ S_T \right]}{K} \right) + \frac{1}{2} B\sigma \right] - K \Phi \left[ \frac{1}{B\sigma} \ln \left( \frac{\mathbb{E} \left[ S_T \right]}{K} \right) - \frac{1}{2} B\sigma \right] - \frac{1}{2}$$

$$= \mathbb{E} \left[ S_T \right] \Phi \left( d_1 \right) - K \Phi \left( d_2 \right),$$

where

$$d_2 = \frac{1}{B\sigma} \ln \left( \frac{\mathbb{E} \left[ S_T \right]}{K} \right) - \frac{1}{2} B\sigma,$$

(13)

and $d_1 = d_2 + B\sigma$. Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula of [Black and Scholes(1973)].
When $\xi_j = (\omega_j, \mu^*_j, \sigma_j, 0)$ for all $j$, where $\mu^*_j = B \left( \mu_j - \frac{1}{2} \sigma_j^2 \right)$ in (3), substituting in expressions (7) and (10) yields, respectively,

$$e^{r\tau} C_t(K; \Lambda) = \sum_{j=1}^{m} \frac{\omega_j \mathbb{E} [ST]}{\Upsilon_j(\Lambda, B)} \Phi (\delta_{1j}) - K \sum_{j=1}^{m} \omega_j \Phi (\delta_{2j}),$$

(14)

where $\delta_{1j}$ and $\delta_{2j}$ are given in (8) and

$$\Upsilon_j(\Lambda, B) = \sum_{l=1}^{m} \omega_l \exp \left\{ B^2 (\mu_l - \mu_j) \right\}.$$

(15)

Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula given in [Bahra(1997)].
When $\xi_j = \left( \frac{1}{m}, \mu, \sigma, \lambda \right)$ for all $j$ in (3), substituting in expressions (7) and (10) yields, respectively,

$$e^{r\tau} C_t(K; \Lambda) = \frac{\mathbb{E}[S_T]}{\Phi(\rho\sigma B)} \int_{-\delta_1}^{\infty} \varphi(z) \Phi[\lambda(z + B\sigma)]dz - K \left[ 1 - F_Y(-\delta_2; \lambda) \right],$$

(16)

where

$$\delta_2 = \frac{1}{B\sigma} \ln \left\{ \frac{\mathbb{E}[S_T]}{2K \Phi(\rho\sigma B)} \right\} - \frac{1}{2} B\sigma,$$

(17)

and $\delta_1 = \delta_2 + B\sigma$. Note that when $B = \sqrt{\tau}$, these expressions coincide with the option pricing formula in [Corns and Satchell(2007)].
The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying stock.

$$\Delta_{\text{Call}} = \sum_{j=1}^{m} \frac{2\omega_j}{\Upsilon_j(\Lambda, B)} \int_{-\delta_{1j}}^{\infty} \varphi(z) \Phi(\lambda_j \tilde{z}_j) \, dz,$$

$$\Delta_{\text{Put}} = \Delta_{\text{Call}} - 1.$$
The gamma of a portfolio of options on an underlying stock is the rate of change of the portfolio’s delta with respect to the price of the underlying stock, i.e., the second partial derivative of the portfolio with respect to stock price

\[
\Gamma_{\text{Call}} = \Gamma_{\text{Put}} = \sum_{j=1}^{m} 2\omega_j \frac{\Phi [-\lambda_j \delta_{2j}]}{\Upsilon_j (\Lambda, B)} \frac{\varphi (\delta_{1j})}{B \sigma_j S}. \tag{18}
\]
GREEK MEASURES

OPTION PRICING BASED ON A LOG-SKEW-NORMAL MIXTURE
by
J. A. JIMÉNEZ, V. ARUNACHALAM, and G. M. SERNA,
DOI:S021902491550051X
We now present an example from real market data to model the distribution of the stock prices and compare the numerical values of a European option under the assumption that the stock movement follow a LSNMIX distribution.

The market data of the S&P500 index for the daily prices were considered from January 4, 2010 to October 13, 2014.
We now present an example from real market data to model the distribution of the stock prices and compare the numerical values of a European option under the assumption that the stock movement follow a LSNMIX distribution.

The market data of the *S&P500* index for the daily prices were considered from January 4, 2010 to October 13, 2014.
The empirical distribution and its descriptive statistics presented in table 1 are analysed using the test proposed by Jarque & Bera (1987), in particular, the skewness and kurtosis, which confirms that the null hypothesis of a normal distribution must be rejected.
### Numerical Results

**Table:** Estimates for adjusting the $SNMIX (\Lambda)$

<table>
<thead>
<tr>
<th>Estimate</th>
<th>MME</th>
<th>MLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>-0.0039</td>
<td>-0.0025</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.0036</td>
<td>0.0045</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0056</td>
<td>0.0056</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.0137</td>
<td>0.0132</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1.4683</td>
<td>1.4686</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-0.6444</td>
<td>-0.6457</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.6158</td>
<td>0.6158</td>
</tr>
</tbody>
</table>
Density functions adjusted for daily returns on S&P 500 Index

**Figure**: Returns vs. normal distribution and SNMIX.
### Numerical Results

<table>
<thead>
<tr>
<th>Strike ( (K) )</th>
<th>( \tau = 0.25 )</th>
<th>( \tau = 0.5 )</th>
<th>( \tau = 0.75 )</th>
<th>( \tau = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td>122,7236</td>
<td>146,2137</td>
<td>168,1804</td>
<td>188,9434</td>
</tr>
<tr>
<td>1350</td>
<td>80,8813</td>
<td>106,9346</td>
<td>130,0796</td>
<td>151,5625</td>
</tr>
<tr>
<td>1400</td>
<td>47,3226</td>
<td>73,8094</td>
<td>96,9744</td>
<td>118,4221</td>
</tr>
<tr>
<td>1450</td>
<td>24,1159</td>
<td>47,8415</td>
<td>69,5369</td>
<td>90,0320</td>
</tr>
<tr>
<td>1500</td>
<td>10,5793</td>
<td>29,0350</td>
<td>47,9058</td>
<td>66,5672</td>
</tr>
</tbody>
</table>

**Table:** Comparison prices of call option Black Scholes.
<table>
<thead>
<tr>
<th>Strike ($K$)</th>
<th>$\tau = 0.25$</th>
<th>$\tau = 0.5$</th>
<th>$\tau = 0.75$</th>
<th>$\tau = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td>123,0441</td>
<td>143,9754</td>
<td>164,5054</td>
<td>184,3572</td>
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<tr>
<td>1350</td>
<td>78,9742</td>
<td>103,5501</td>
<td>125,8430</td>
<td>146,8139</td>
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<td>1400</td>
<td>44,7446</td>
<td>70,3303</td>
<td>92,9945</td>
<td>114,1851</td>
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<td>1450</td>
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<td>45,2095</td>
<td>66,5457</td>
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<tr>
<td>1500</td>
<td>10,7043</td>
<td>28,0958</td>
<td>46,5603</td>
<td>65,1000</td>
</tr>
</tbody>
</table>

**Table:** Comparison prices of call option Corrado & Su.
### Numerical Results

<table>
<thead>
<tr>
<th>Strike ($K$)</th>
<th>Maturities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau = 0.25$</td>
</tr>
<tr>
<td>1300</td>
<td>121,8372</td>
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<tr>
<td>1350</td>
<td>76,7124</td>
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<td>1400</td>
<td>35,8423</td>
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<tr>
<td>1450</td>
<td>8,3470</td>
</tr>
<tr>
<td>1500</td>
<td>1,6477</td>
</tr>
</tbody>
</table>

**Table:** Comparison prices of call option SNMIX.
**Figure:** Call option for different strikes.
**FIGURE**: Call option for different maturities.
**Figure:** Implied volatility.
Conclusions

We have proposed an alternative approach for calculating the price of the call and put options when the stock return distribution follows the SNMIX distribution.

- We have obtained an explicit expression for the price of the European options, and from the proposed model, we deduce some well-known models, such as [Black and Scholes(1973)], [Bahra(1997)] and [Corns and Satchell(2007)].

- An example from real market data is also presented to implement the proposed model, and comparisons are made with well-known models.

- The derivation of an expression for the implied volatilities at and around-the-money and studies of its asymptotic behaviour are in progress.
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THANK YOU VERY MUCH
Conclusions


Bhupinder Bahra. Implied risk-neutral probability density functions from option prices: A central bank perspective.
Dimitris N. Politis.
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R. Jarrow and A. Rudd.
Approximate option valuation for arbitrary stochastic processes.

Charles J. Corrado and Tie Su.
Skewness and kurtosis in s&p 500 index returns implied by option prices.
Implied volatility skews and stock index skewness and kurtosis implied by S&P 500 index option prices.


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Conclusions


