Capital Valuation Adjustment and Funding Valuation Adjustment

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BASED ON

- C. Albanese, S. Caenazzo and S. Crépey, Capital valuation adjustment and funding valuation adjustment, arXiv and ssrn (updated version available upon request at screpey@gmail.com)
  - Abridged version Capital and funding published in *Risk Magazine* May 2016
- See also Green and Kenyon KVA papers and the Solvency 2 actuarial literature

Outline

CVA Credit valuation adjustment

- The value you miss due to the defaultability of your counterparty

DVA Debit valuation adjustment

- The value your counterparty misses due to your own defaultability
- The symmetric companion of the CVA
- The value you gain due to your own defaultability? (2011 DVA debate)

FVA Funding valuation adjustment (cost of funding variation and initial margin, in other words I am putting the MVA into it to spare one "VA")

- But what about the Modigliani-Miller theorem?? (2013 FVA debate)

DVA2 Funding windfall benefit at own default

KVA Cost of capital

- The price of market incompleteness for banks (2015 KVA debate)

Total VA=CVA−DVA+FVA−DVA2 [+KVA]: The XVA debates

- FVA and DVA2 cash flows NPV-match each other
- Accordingly, CVA-DVA yields the fair, symmetrical adjustment between two counterparties of equal bargaining power
- But contra-liability DVA and DVA2 cash flows are only a benefit to the creditors of the bank, whereas only the interest of shareholders should matter in bank's managerial decisions, hence DVA and DVA2 should be ignored in entry prices
- However, counterparty credit risk markets are incomplete and banks's shareholders must reserve capital to cope with residual risk
- Capital deserves a remuneration at a hurdle rate, which corresponds to the last KVA term
- In practice, banks are price makers, end-clients are price takers and banks can only survive by passing to the end-clients the whole XVA suite
- Hence, from a price maker perspective where contra-liability cash flows are disregarded, the risk-neutral total VA is CVA+FVA and the risk-adjusted total VA is FTP=CVA+FVA+KVA

- The Modigliani-Miller theorem is.. a theorem, not an axiom!
- Quoting the conclusion of their seminal 1958 paper

  > *These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share.*

- And Miller (1988) in "The Modigliani-Miller Proposition after Thirty Years"

  > *Showing what doesn't matter can also show, by implication, what does.*

More specifically, quoting Villamil (2008):

> *In fact what is currently understood as the Modigliani-Miller Proposition comprises four distinct results from a series of papers (1958, 1961, 1963). The first proposition establishes that under certain conditions, a firms debt-equity ratio does not affect its market value. The second proposition establishes that a firms leverage has no effect on its weighted average cost of capital (i.e., the cost of equity capital is a linear function of the debt-equity ratio). The third proposition establishes that firm market value is independent of its dividend policy. The fourth proposition establishes that equity-holders are indifferent about the firms financial policy.*

- The proof of the fourth proposition is based on the ability of shareholders to redeem all debt of the bank in order to prevent wealth transfers to creditors.

However:

- Banks are special firms in that they are intrinsically leveraged and cannot be transformed into a pure equity entity.
- This is also related to an argument of scale.
- Banks liabilities are overwhelming with respect to all other wealth numbers.
- It has been estimated that if all European banks were to be required to have capital equal to a third of liabilities, the total capitalization of banks would be greater than the total capitalization of the entire equity market as we know it today.
  - In other words, only bank stocks would be traded and other firms would have nothing left for them.

Hence:

- Shareholders cannot redeem all debt of the bank.
- The (assumption of the) fourth proposition of the Modigliani-Miller theorem does not apply to a bank.
- The interest of bank shareholders and creditors are not aligned to each other

Outline

- There is common agreement that the KVA should be something like $h\mathbb{E}\int_0^T C_t dt$ where h is some hurdle rate, $\mathbb{E}$ is average/expectation and C_t is capital at risk at future time t
- If yes, why do we need to talk more about it?
  - Depending on the nature of the KVA, needs to be included in financial reporting or not
  - Still room for a factor of 30% to 300% depending on the detailed specification of KVA
    - $\mathbb{P}$ vs. $\mathbb{Q}$
    - Which discounting
    - Which hurdle rate
    - Regulatory/economic capital (EC=some risk measure of mis-hedge losses)
    - Model risk

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- Is it a price in the sense of the cost of replication?
  - Is "$hC_t dt$" a cash-flow in the first place?
  - "C_t" is capital earmarked to absorb exceptional unhedged losses
  - $h\int_0^T C_s ds$ is quite volatile and there is no reasonable way one can "hedge" this out
- Is it a price in the sense of no arbitrage valuation, modulo some hedging error?
  - What then about "KVA un-hedged losses", where un-hedged losses in general are precisely meant to be covered by... economic capital / KVA?
- Is it a price in the sense of a risk premium (like for a stock)?
  - Counterparty credit risk (CCR) hard to hedge in practice
  - $hC_t dt \sim$ Average$_t$ KVA cash-flow
  - "Average$_t$ shareholders' return $=$ risk aversion $\times$ risk$_t$"
  - Derivative portfolio optimization theory for a derivative market-maker in incomplete derivative markets
    - Intrinsic incompleteness related to counterparty credit risk
  - A well-understood KVA puts the bank back on the efficient (and also prudent..) frontier
  - Bank shareholders risk premium (even if $\mathbb{P} = \mathbb{Q}$)

- Any precedents?
  - Solvency 2 “risk margins”, i.e. cost of capital
  - An old classic, cf. Modigliani and Miller 1958 paper again:
    “ The cost of capital , corporate finance and the theory of investment”
  - Insurers deal with financial risk at the portfolio level on a risk margin basis, as opposed to individual hedge of derivatives in the banking tradition of complete markets
- Can a bank really learn from Solvency 2?
  - Is a bank entitled to do it or would that be a regulatory mismatch?
  - Even under the current banking regulatory environment, why would it not be possible for a bank to decide to implement a cost of capital KVA strategy on a voluntary basis, even without a prescriptive regulatory environment, as a way to implement a sustainable dividend distribution policy?

If the banking KVA should just be the banking analog of Solvency 2 risk margins, why do we need to talk more about KVA?

- Insurers have a lot of liquidity
- While insurance portfolios have only a KVA-like metric called risk margin, banks have several other XVA metrics
  - In particular, economic capital and the banking FVA metric are intertwined together Go
- In fact, insurance may even have to learn from banks regarding KVA/risk margins
  - Truly dynamic (not only multi-period) perspective

Outline

To size the KVA we ask the following question:

What should be the size of a KVA account (retained earnings) ending up at zero at the final maturity of the portfolio, assumed held on a run-off basis, and such that one can remunerate bank shareholders from this account an average/expected "$h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$" at any future point in time?

The reason why "$(\mathrm{EC}_t - \mathrm{KVA}_t)$" and not "$\mathrm{EC}_t$" here is because KVA is loss-absorbing, hence shareholders capital at risk is only the difference between EC_t and KVA_t

- Why should the KVA *not* be loss-absorbing?
- Solvency 2 risk margins are loss-absorbing
- KVA commonly represents in the order of about one-half of EC, excluding it from economic capital would mean significantly more capital at risk required from the shareholders, which would be very expensive

Go

“..ending up at zero at the final maturity of the portfolio (or the default of the bank itself)”

Less than zero would be insufficient and more would be wasteful

“..the portfolio, assumed held on a run-off basis..”

- In order to ensure a dividend policy that would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future

Go

- Prudent valuation
- If future trades could be anticipated, then one could optimize further and assess the fair valuation of debt ahead of time to compensate for the anticipated occurrence of wealth transfers at future deals.
  - Cf. John Hull’s April 2016 XVA Risk piece
  - This would lead to greater efficiencies and allow banks to bid for trades at entry prices given by unadjusted fair valuations.
- However, the impossibility of anticipating new trades and otherwise offsetting wealth transfers, motivates us to work in the conservative limit whereby incremental XVAs are computed under the run-off assumption.
  - Asymmetrical information versus clairvoyance
  - A price-maker has no crystal ball

"average/expected"

- Shareholders want their $h = 10\%$ average return on capital at risk under $\mathbb{P}$, not $\mathbb{Q}$
- However, in the context of optimization theory for a derivative market-maker, the main source of information is market prices of liquid instruments, which allow one to infer a pricing measure $\mathbb{Q}$
- While there is little relevant that can be said about the historical probability measure $\mathbb{P}$
- Hence, $\mathbb{P} = \mathbb{Q}$ may be in the end the "less arbitrary modeling assumption"

"economic/regulatory capital EC_t"

- For KVA computations entailing capital projections over decades, an equilibrium view based on Basel II Pillar II economic capital is more attractive than the ever-changing Pillar I regulatory charges supposed to approximate it
- Using regulatory instead of economic capital is motivated by practical considerations but is less self-consistent
  - Loses the connection where the KVA input should be the the loss-and-profit ($-P\&L$) process L of the derivative portfolio.
  - Losses arising from CCR and other XVAs mis-hedge in particular

- Without "average(=expected) return" in "the KVA question", the answer would be that an infinite amount has to be retained for the KVA
  - Unless a corresponding hedge is setup, but this would involve a circularity as said above
  - The KVA problem would be ill-posed.
- Thanks to the "average(=expected) return", the KVA problem is well posed
  - The actual dividend(/dilution) to shareholders amounts to $-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt)$, with drift assigned to be $h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$
  - The KVA process needs to satisfy $\mathrm{KVA}_T = 0$ and $d\mathrm{KVA}_t + h(\mathrm{EC}_t - \mathrm{KVA}_t)dt - r_t\mathrm{KVA}_t dt = dM_t$, for some martingale M.
  - Unique solution given as $\mathrm{KVA}_t = h\mathbb{E}\int_t^T e^{-\int_t^s (r_u+h)du}\mathrm{EC}_s ds$
  - The "$+h$" in the discount factor reflects the "$-\mathrm{KVA}_t$" in "$h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$"
    - Fact that the KVA is loss-absorbing

Solvency constraint $\mathrm{SCR}_t = \mathrm{EC}_t - \mathrm{KVA}_t \geq 0$

- "Solvency accounting condition"
- Otherwise the KVA would cease to increase in the hurdle rate h and to be a supermartingale
- Set aside a bit more economic capital so that the ensuing KVA is below it
- KVA backward SDE (BSDE)
- This said, the constraint is immaterial unless the term structure of EC starts very low and has a sharp peak in a few years
  - Quite uncommon for a derivative portfolio assumed held on a run-off basis, where the bulk of the portfolio consists of trades with 3y to 5y maturity

Go

Outline

Putting the KVA on the right hand side (liabilities) of the balance sheet would be erroneous conceptually..

- KVA already on the asset side of the balance sheet as retained earnings
- Having it on the liability side as a capital deduction would be strange given that the KVA is loss-absorbing
- KVA is NOT the no arbitrage valuation of some cash-flow
- KVA $\neq$ in nature from CVA and FVA
- KVA $=$ cost of capital required to cope with CCR (including other VAs) mis-hedge
- It is the balance-sheet (fluctuations) that should serve as input to the KVA and not the other way around

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- Accounting for the FVA in the balance sheet (as 2/3 banks already do) is good accounting practice
  - Current accounting principles are based on a complete market mindset and need to be revised
- The KVA on the other hand should NOT be accounted for in the balance sheet as this would be circular and nonsensical.
- The KVA contributes to entry prices but not to the P&L itself (trading P&L from the shareholders viewpoint), it is the shareholders' risk premium deserved by the fluctuations of the P&L

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..not also flawed conceptually, but also makes it intractable numerically, unless further approximations are made

- Ignoring that the KVA is loss-absorbing, i.e. simplifying $h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$ (the expected $-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt)$ cash-flow, assuming $\mathbb{P} = \mathbb{Q}$) into $h\mathrm{EC}_t dt$
  - Otherwise the KVA itself would enter the all-inclusive XVA equation
- Working with regulatory instead of economic capital, which loses self-consistency
  - Otherwise forward starting one-year-ahead fluctuations of the KVA should be simulated for capital and in turn KVA calculation. This would both involve a conceptual circularity and be intractable numerically.

Starting from the wrong premise and using these approximations, one obtains a single, KVA-inclusive XVA equation, with one additional $h\mathrm{EC}_t dt$ term on top of the CVA and FVA terms.

Starting from the wrong premise and using these approximations, one obtains a single, KVA-inclusive XVA equation, with one additional $h\mathrm{EC}_t dt$ term on top of the CVA and FVA terms.

- The ensuing KVA is "too high" because it misses the required "$+h$" in the discount rate
- This may lead to paradoxical situations where the KVA is greater than the capital it is issued from
- This is partially mitigated by the fact that this KVA is (unduly) computed with regulatory capital, which is in principle a lower bound on economic capital

However, why should one do that??

- The "right" KVA equation, on top of being well-grounded economically, does not require the distorting approximations of the previous slides
- It never leads to KVA$>$ EC.

Outline

XVA do not revolutionize math finance, they only revisit it

- In the end only three XVAs really matter (impact shareholders / should impact entry prices):
  - CVA, i.e. what remains of the portfolio after a back-to-back hedge has been setup
  - FVA, has to do with funding (of variation and initial margins)
  - KVA, as to do with hedging (error)
- Hence, we recognize the three components of a trading strategy that math finance has always considered traditionally (the three traditional pillars of price and hedge analysis)
  - The derivative portfolio, its funding policy, and its hedge (or hedging error)

- KVA corresponds to the risk of the bank viewed on the equity side of the balance sheet
  - The asset side/financial perspective on risk measures
  - Not as much how much to remove from my liabilities to be safe but how much to remove from my assets to be in trouble
- Also AVA ("additional" $\leftrightarrow$ prudent valuation, accounting for model risk)
  - Which $\mathbb{Q}$
  - Which $\mathbb{P}$ vs. $\mathbb{Q}$ discrepancy
  - Model risk (call it AVA or not) should inflate even more the importance of KVA (which is driven by extremes) with respect to other XVAs

Outline

Implementation at the portfolio level

Go

- Doable using the appropriate large memory (better than clouds) and GPU grid technologies..
  - On a representative bank portfolio the whole calculation takes a few minutes for building the models, followed by a nested simulation time in the order of about an hour per billion (primary and secondary) scenarios Go
  - Nested Monte Carlo
- .. and mathematics (models and algorithms)
  - Only based on in-memory compliantly parallelizable algorithms
  - Products of matrices in single-float precision arithmetics
  - Markov chains models with regimes

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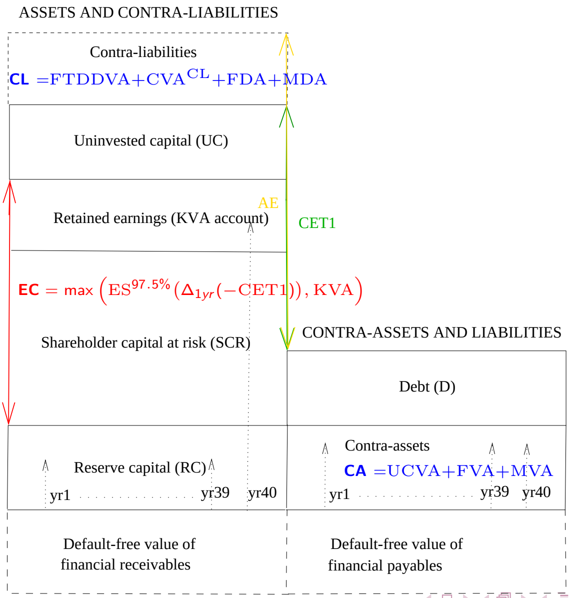
TECHNICAL PRESENTATION

Introduction

- In the aftermath of the financial crisis, regulators launched in a major banking reform effort aimed at securing the financial system by raising collateralisation and capital requirements, as if the costs of capital and of funding for collateral were irrelevant.
- The quantification by banks of market incompleteness based on various XVA metrics, in particular KVA (capital valuation adjustment) and FVA (funding valuation adjustment), is emerging as the unintended consequence of the banking reform.
- The presence of the KVA and the FVA breaks several of the conclusions of Modigliani-Miller theory.
- In this paper we reconsider XVAs in the light of a structural model of the bank.
- In order to focus on counterparty credit risk and XVAs, we assume that the market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades.

Outline

Model of the Balance Sheet of a Bank



A, L: Assets (A) and liabilities (L), are computed ignoring counterparty credit risk. In the assets (liabilities) accounts, one places, in particular, default free valuation of all unsecured derivative receivables and derivative payable hedges (unsecured derivative payables and derivative receivable hedges).

CA: In the contra-assets (CA) account, one places model valuations of future cash-flow streams related to counterparty credit risk and having an impact on shareholder value.

CL: In the contra-liabilities (CL) account, one places all other metrics assessing model valuations of future cash-flow streams related to counterparty credit risk and having an impact on bank creditors, by increasing the recovery rate of the bank, but not on shareholders.

- Standing example: The **DVA** (debt valuation adjustment)

Quoting http://www.accountingtools.com/questions-and-answers/what-is-a-contra-asset.html:

> *The proper size of a contra asset account can be the subject of considerable discussion between a company controller and the company's auditors. The auditors want to ensure that reserves are adequate, while the controller is more inclined to keep reserves low in order to increase the reported profit level. The resulting contra asset balances can be negotiated figures.*

And http://www.accountingtools.com/questions-and-answers/what-is-a-contra-liability-account.html:

> *If the amount in a contra liability account is immaterial, it could reasonably be combined into a single balance sheet line item with the liability that it is intended to offset. Or, if the contra liability account balance is immaterial, the accounting staff could elect not to keep a balance in the account at all. In practice, contra liability accounts are rarely used.*

The above-mentioned issues of the valuation of contra-assets and of the materiality of contra-liabilities, in the particular case of a bank, are at the core of this paper.

Definition 1

The difference between the left side, including (resp. excluding) contra-liabilities, and the right side of the balance sheet yields the accounting equity AE (resp. the core equity tier I capital CET1) of the bank, i.e.

$$\mathrm{AE} = \mathrm{A} - (\mathrm{L} + (\mathrm{CA} - \mathrm{CL})), \quad (1)$$

$$\mathrm{CET1} = \mathrm{A} - (\mathrm{L} + \mathrm{CA}). \quad (2)$$

Lemma 1

- AE *represents the wealth of the bank as a whole.*
- $\mathrm{CET1}$ *represents the wealth of bank shareholders.*

Proof. The wealth of bank shareholders is the wealth of the bank net of its contra-liabilities, i.e. the CET1. ∎

- The specification of different CA and CL components is set-up dependent.
- Their detailed formulation is actually the goal of a CCR pricing model

Unless CL vanishes, Lemma 1 breaks the conclusion of the Modigliani and Miller (1958) theorem according to which the interest of shareholders and creditors are aligned.

- Quoting Villamil (2008):

> *In fact what is currently understood as the Modigliani-Miller Proposition comprises four distinct results from a series of papers (1958, 1961, 1963). The first proposition establishes that under certain conditions, a firms debt-equity ratio does not affect its market value. The second proposition establishes that a firms leverage has no effect on its weighted average cost of capital (i.e., the cost of equity capital is a linear function of the debt-equity ratio). The third proposition establishes that firm market value is independent of its dividend policy. The fourth proposition establishes that equity-holders are indifferent about the firms financial policy.*

- The proof of the fourth proposition is based on the ability of shareholders to redeem all debt of the bank in order to prevent wealth transfers to creditors.
- However, banks are special firms in that they are intrinsically leveraged and cannot be transformed into a pure equity entity.
- Hence, this part of the Modigliani and Miller (1958) theorem does not apply to a bank.

Dynamic Setup

- We find it helpful to introduce the concept of a "CA desk" of the bank, which would sell the contra-assets to the clients of the bank and then be exposed to the corresponding payoffs
  - Counterparty default losses and funding expenditures
- The CA desk may also setup a CA hedge, i.e. a hedge of these payoffs.

Definition 2

We call CA desk loss process L the difference between, on the one hand, the counterparty default losses and funding expenditures and, on the other hand, the depreciation of the contra-assets, minus the CA hedging gain (starting from the accrued loss $L_0 = y$, which is the negative of the initial endowment of the portfolio).

- Hence, the process L can be interpreted as the loss(-and-profit) process of the CA desk.
  - The CA desk loss process L should not be confused with the CA value process, which represents the model valuation of the contra-assets.
- The precise specification of the processes L and in turn CA is in fact the goal of a counterparty credit risk pricing model.

- The derivative portfolio of the bank is modelled on a run-off basis until its final maturity T (or the bank's default time τ).
  - Note that we refrain from treating states of zero or negative equity as triggers of bank default as in Andersen, Duffie, and Song (2016, Sect. III-IV) because re-capitalisation decisions, possibly due to government intervention, empirically play a major role to stave off defaults. Instead we model the default of the bank as a totally unpredictable event at some exogenous time τ.
  - In the same line of thinking, in case exceptional loss events occur of severity typical of the worst event once every 40 years, according to the total loss-absorbing capacity standard TLAC (see Financial Stability Board (2015)), debt is converted into equity to replenish CET1. This conversion would instantaneously replenish both shareholder capital at risk and the KVA (so that we do not need to keep track of the times of exceptional losses in our model).

- Losses-and-earnings realization times are typically quarter ends for bank profits, released as dividends, vs recapitalization managerial decision times for losses. However, there is no way to calibrate losses-and-earnings realization times in a pricing or risk model. In our framework, we assume that losses-and-earnings are marked to model and realized in real time.
- In particular, the gains $(-dL_t)$ continuously flow into realized profit-and-losses. Equivalently, the RC account is assumed to be continuously reset to its theoretical target CA level: Much like with futures, the position of the CA desk is reset to zero at all times, but it generates gains $(-dL_t)$.
- Similarly, retained earnings are continuously reset to their theoretical target KVA level (which, within our proposed treatment for retained earnings in this paper, will be given as the solution to the KVA BSDE (9) below). As a consequence, $(-d\mathrm{KVA}_t)$ amounts continuously flow from the KVA account to the shareholders, who algo get $r_t \mathrm{KVA}_t dt$ from an OIS accrual of the KVA account.

Lemma 2

Shareholders dividends net to

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt). \tag{3}$$

Whenever negative, this is interpreted as recapitalization, i.e. equity dilution.

Proof. Under our back-to-back market hedge assumption on the portfolio, the trading gain process of the bank, from the point of view of the shareholders, reduces to $(-L)$ (cf. the balance-sheet). Also accounting for released earnings and OIS accrual payments from the KVA account, shareholders dividends net to (3). ∎

The practical consequence of the next result will be that the CA desk loss process L can be used as the bank shareholders loss process instead of $(-\mathrm{CET1})$ for any XVA pricing or risk modelling purpose ($\rightarrow$ "reduced model" L instead of CET1).

Proposition 1

Under our standing modeling assumptions of a back-to-back hedged portfolio held on a run-off basis and under a TLAC resolution scheme of exceptional losses (with losses-and-earnings marked to model and realized in real time), it holds that

$$L_t = L_0 + \mathrm{CET1}_0 - \mathrm{CET1}_t, \tag{4}$$

where L is the CA desk loss process. Consequently:

(i) *The absence of shareholders arbitrage, otherwise said CET1 being the fair valuation of shareholders' capital, i.e. the existence of a risk-neutral measure $\mathbb{Q}$ under which the process CET1 is a risk-neutral local martingale, is equivalent to the analog risk-neutral local martingale property regarding L, i.e. to fair valuation of target reserve capital.*

(ii) *The time fluctuations of the process $(-\mathrm{CET1})$ are nothing but the time fluctuations of the process L.*

Proof. We have (cf. the balance-sheet)

$$\begin{aligned}\mathrm{CET1} &= \mathrm{A} - \mathrm{L} - \mathrm{CA}\\ &= (\mathrm{SCR} + \mathrm{KVA} + \mathrm{UC} - \mathrm{D}) + (\mathrm{RC} - \mathrm{CA}),\end{aligned} \tag{5}$$

where:

- For a portfolio held on a run-off basis and under a TLAC resolution scheme, the different entries in the first parenthesis affect each other like interconnecting pipes;
- Under mark-to-model and realization of losses-and-earnings in real time, the two terms in the second parenthesis in (5) constantly match each other, but they generate the non-vanishing gain process $(-L)$, where L is the CA loss process as of Definition 2. ∎

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Incremental XVA Methodology

- XVA numbers are computed assuming the portfolio held on a run-off basis in order to ensure a dividend policy that would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future
- This is part of prudent valuation
- If future trades could be anticipated, then one could optimize further and assess the fair valuation of debt ahead of time to compensate for the anticipated occurrence of wealth transfers at future deals.
  - Cf. Hull's April 2016 XVA Risk piece
  - This would lead to greater efficiencies and allow banks to bid for trades at entry prices given by unadjusted fair valuations.
- However, the impossibility of anticipating new trades and otherwise offsetting wealth transfers, motivates us to work in the conservative limit whereby incremental XVAs are computed under the run-off assumption.
  - Asymmetrical information versus clairvoyance
  - A price-maker can have no crystal ball

- Even though portfolios are modelled as if they were held on a run-off basis, time 0 in the model represents the time a new deal is considered and for which incremental XVA amounts need to be computed and charged to the client.
- The total add-on to the entry price for a new deal is called funds transfer pricing (FTP).
- We denote with (without) a "$\widetilde{\cdot}$" all the previously introduced quantities regarding the portfolio including (without) the new deal and its back-to-back hedge and we denote by $\Delta\cdot$ any incremental amount $\cdot - \widetilde{\cdot}$.

Proposition 2

Under a static no arbitrage assumption that the wealth of the shareholders should not jump upon inclusion of the new deal apart from a $\Delta\mathrm{KVA}$ amount used for updating retained earnings in accordance with the new deal, then the total XVA charge passed on to the client of a deal must be given by the FTP formula

$$\mathrm{FTP} = \Delta\mathrm{CA} + \Delta\mathrm{KVA}. \tag{6}$$

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Proof. In case a new deal is traded, the difference between the gross shareholder capital of the bank (shareholders' capital at risk plus uninvested equity capital) and its debt remains constant

- For instance, if more debt is issued to fund a trade, the corresponding cash goes into assets, which therefore increase by the same amount).

Hence, in case a new deal is traded, one has (cf. the balance-sheet):

$$\Delta(\mathrm{A} - \mathrm{L}) = \Delta\mathrm{RC} + \Delta\mathrm{KVA}. \tag{7}$$

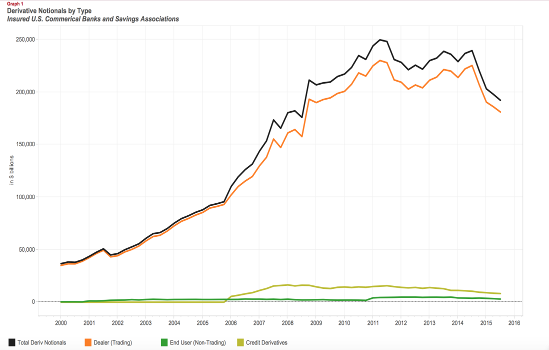
Therefore, in view of (2),

$$\Delta(\mathrm{CET1} - \mathrm{KVA}) = \Delta\mathrm{RC} - \Delta\mathrm{CA}. \tag{8}$$

The stated assumption reads $\Delta(\mathrm{CET1} - \mathrm{KVA}) = 0$, which is then equivalent to $\Delta\mathrm{RC} = \Delta\mathrm{CA}$. Accounting for the $\Delta\mathrm{KVA}$ amount required from the client in order to update the level of retained earnings in accordance with the new deal, this results in the FTP formula (6). ∎

Outline

Ponzi scheme in the last financial crisis (source: OCC Q3 2015 Quarterly Bank Trading Revenue Report).



We introduce a cost of capital framework for assessing KVA, pass it on to the bank's clients and distribute it gradually to the bank's shareholders through a dividend policy which would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future.

Return

- Let $\mathrm{ES} = \mathrm{ES}_t(L) = \mathrm{ES}_t^{97.5\%}\big(-\Delta_{1yr}\mathrm{CET1}\big)$ denotes the 97.5% conditional expected shortfall of the one-year-ahead reserve capital loss $(L_{t+1} - L_t)$.
- Let $C \geq \mathrm{ES}$ represent a putative economic capital process for the bank.
- To size the KVA we ask the following question: What should be the level $K = K_t(C)$ of a KVA account ending up with $K_{\bar{\tau}} = 0$ (less would be insufficient and more would be wasteful) and such that the ensuing dividend stream

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt)$$

(cf. Lemma 2) corresponds to an average (expected) remuneration $h(C_t - K_t)dt$ at each point in time $t \in [0, \bar{\tau}]$ to the shareholders?
- The reason why $(C_t - K_t)$ rather than C_t here is that retained earnings are loss-absorbing and therefore part of the economic capital. Therefore, shareholders' capital at risk, which by assumption is remunerated at the hurdle rate h, only corresponds to the difference $(C_t - K_t)$.
- Solvency accounting constraint: The difference $(C_t - K_t)$ represents shareholder capital at risk and must therefore be non-negative.

Lemma 3

Consider the following backward stochastic differential equations (BSDEs for short, see El Karoui, Hamadène, and Matoussi (2009) for a survey):

$$K_t = \mathbb{E}_t \int_t^{\bar{\tau}} \Big(hC_s - (r_s + h)K_s \Big) ds\,, \quad t \in [0, \bar{\tau}], \tag{9}$$

$$\mathrm{KVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \big(h \max(\mathrm{ES}_s, \mathrm{KVA}_s) - (r_s + h)\mathrm{KVA}_s \big) ds\,, \quad t \in [0, \bar{\tau}] \tag{10}$$

to be solved for respective processes K and KVA. Assuming that r is bounded from below and that C (respectively ES) and r are in $\mathcal{H}^2$, then the BSDE (9) *(respectively* (10)*) is well posed in $\mathcal{H}^2$, where well-posedness includes existence, uniqueness, comparison and the standard a priori bound and error BSDE estimates.*
The $\mathcal{H}^2$ solution K to (9) *admits the explicit representation*

$$K_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u + h) du} C_s ds,\ t \in [0, \bar{\tau}]. \tag{11}$$

Proof. Monotonic coefficient BSDEs (in a general filtration) + linear BSDE (9). ∎

Definition 3

(i) The set of admissible economic capital processes is

$$\mathcal{C} = \{C \in \mathcal{H}^2; C \geq \max(\mathrm{ES}, K(C))\}, \tag{12}$$

where $C \geq \mathrm{ES}$ is the risk acceptability condition and $C \geq K(C)$ is the Solvency-like accounting condition (cf. the respective conditions (b) and (a) and their discussion in Wüthrich and Merz (2013, pages 270 and 271)).
(ii) Let

$$\mathrm{EC} = \max(\mathrm{ES}, \mathrm{KVA}) \tag{13}$$

in $\mathcal{H}^2$, where KVA is the $\mathcal{H}^2$ solution to the BSDE (10) (assuming ES in $\mathcal{H}^2$).

Remark 1

In many cases as in the figure 78, we have that $\mathrm{EC} = \mathrm{ES}$. The inequality stops holding when the hurdle rate is high enough and the term structure of EC starts very low and has a sharp peak in a few years.

Lemma 4

Assuming that r is bounded from below and that ES and r are in $\mathcal{H}^2$, the solution KVA to (10) solves the linear BSDE (9) for the implicit data $C = \mathrm{EC}$, i.e. we have $\mathrm{KVA} = K(\mathrm{EC})$ or, equivalently,

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \mathrm{EC}_s ds,\ t \in [0, \bar{\tau}]. \tag{14}$$

In particular, the KVA process discounted at the OIS rate is a supermartingale.

Proof. BSDE verification + uniqueness. ∎

Theorem 1

Under the assumptions of Lemma 4, we have:
(i) $\mathrm{EC} = \min \mathcal{C}, \mathrm{KVA} = \min_{C \in \mathcal{C}} K(C)$.
(ii) *If* $\mathrm{ES} \geq 0$, *then* KVA *is nondecreasing in* h.

Proof. BSDE verification + uniqueness. BSDE comparison theorem. ∎

Outline

Trading Setup

- We assume that the bank is engaged into bilateral trading of a derivative portfolio split into several netting sets corresponding to counterparties indexed by $i = 1, \ldots, n$.
- Let MtM^i_t be the mark-to-market of the i-th netting set, i.e. the trade additive risk-neutral conditional expectation of future discounted promised cash flows, ignoring counterparty credit risk and funding costs.
- Let
$$P^i_t = \mathrm{MtM}^i_t - \mathrm{VM}^i_t \tag{15}$$
be the net spot exposure of the i-th netting set, i.e. the difference between the mark-to-market of the netting set and the corresponding (algebraic) variation margin VM^i_t received by the bank.
- In addition to the variation margin VM^i_t that flows between them, the counterparty i and the bank post respective initial margins IM^i and $\overline{\mathrm{IM}}^i_t$ in some segregated accounts.
- Finally, we denote by R_i the unsecured borrowing recovery rate of the counterparty i.

- In practice, there is a positive liquidation period, usually a few days, between the default of a counterparty (or the bank) and the liquidation of its portfolio.
- Once a position is fully collateralized in terms of variation margin, the gap risk related to the slippage of MtM^i_t and to unpaid cash-flows during the liquidation period becomes the first order residual risk and the motivation for the initial margins.
- A positive liquidation period is explicitly introduced in Armenti and Crépey (2015) and Crépey and Song (2016) (see also Brigo and Pallavicini (2014)) and involves introducing the random variables:

$$\mathrm{MtM}^i_{\tau_i+\delta t} + \delta t\mathrm{MtM}^i_{\tau_i+\delta t} - \mathrm{VM}^i_{\tau_i}, \tag{16}$$

where δt is the length of the liquidation period and $\delta t\mathrm{MtM}^i_{\tau_i+\delta t}$ is the accrued value of all the cash flows owed by the counterparty of the bank during the liquidation period.

- To alleviate the notation in this work, we take the limit as $\delta t \to 0$ and approximate $\mathrm{MtM}^i_{\tau_i+\delta t} + \delta t \mathrm{MtM}^i_{\tau_i+\delta t}$ through $\widehat{\mathrm{MtM}}^i_{\tau_i}$, and therefore (16) through $Q^i_{\tau_i} = \widehat{\mathrm{MtM}}^i_{\tau_i} - \mathrm{VM}^i_{\tau_i}$, for suitable $\mathbb{G}$-optional processes $\widehat{\mathrm{MtM}}^i$ and $Q^i = \widehat{\mathrm{MtM}}^i - \mathrm{VM}^i$.
- Another related issue is wrong-way risk, i.e. the adverse dependence between the CVA exposure of the bank and the credit risk of its counterparties. This impact can also be captured in the random variable $Q^i_{\tau_i}$ (see Crépey and Song (2016)).

Lemma 5

The exposure of the bank to the default of each counterparty $i = 1, \ldots, n$ is

$$(1 - R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+ \tag{17}$$

Proof. Let $C^i = \mathrm{VM}^i + \mathrm{IM}^i$ and

$$\epsilon_i = (Q^i_{\tau_i^\delta} - \mathrm{IM}^i_{\tau_i})^+ = (\widehat{\mathrm{MtM}}^i_{\tau_i^\delta} - C^i_{\tau_i})^+.$$

Also accounting for the back-to-back hedge of the netting set i which is unwound at the time of liquidation of the counterparty i, the loss of the bank in case of default of the counterparty i is given as

$$\begin{aligned} &\widehat{\mathrm{MtM}}^i_{\tau_i^\delta} - \mathbb{1}_{\epsilon_i=0}\widehat{\mathrm{MtM}}^i_{\tau_i^\delta} - \mathbb{1}_{\epsilon_i>0}(C^i_{\tau_i} + R_i\epsilon_i) \\ &= \mathbb{1}_{\epsilon_i>0}(\widehat{\mathrm{MtM}}^i_{\tau_i^\delta} - C^i_{\tau_i} - R_i\epsilon_i) = (1 - R_i)\epsilon_i. \blacksquare \end{aligned} \tag{18}$$

- Symmetrically, the exposure of each counterparty $i = 1, \dots, n$ to the default of the bank is given as

$$(1-R)(Q^i_\tau - \overline{\mathrm{IM}}^i_\tau)^-, \tag{19}$$

where R is the unsecured borrowing recovery rate of the bank.
    - The DVA originating cash-flow of the bank at its own default time τ arises as

$$(1-R)\sum_{i\neq 0} J^i_\tau (Q^i_\tau - \overline{\mathrm{IM}}^i_\tau)^-. \tag{20}$$

- The regulator says quite explicitly that a firm capital cannot be seen increasing as a consequence of the sole deterioration of the bank credit, all else being equal.
- Accordingly, it requires banks to deduct from CET1 the unilateral UCVA pricing the cash-flows valued by the CVA over an infinite time horizon
    - As opposed to the first-to-default FTDCVA pricing the cash-flows valued by the CVA until the first default time of the bank and each considered counterparty
- As will be discussed below, this spoils the financial meaning of CET1 as being the fair valuation of shareholders' capital, but we can restore this meaning by postulating that, at the default time of the bank τ, a fictitious wealth transfer of amount

$$\mathrm{UCVA}_\tau = \mathbb{E}_\tau \sum_{\tau < \tau_i} \beta_\tau^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+ \tag{21}$$

CA and FVA BSDEs

Lemma 6

In the case of a portfolio of bilateral trades between the bank and n counterparties $i = 1, \dots, n$, we have:

$$
\begin{aligned}
&L_0 = y \text{ and, for } t \in (0, \bar{\tau}], \\
&dL_t = d\mathrm{CA}_t + \sum_{i \neq 0} (1 - R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+ \delta_{\tau_i}(dt) + \mathrm{UCVA}_\tau \delta_\tau(dt) \\
&+ \Big(\lambda_t \big(\sum_{i \neq 0} J^i_t P^i_t - \mathrm{EC}_t(L) - \mathrm{CA}_t\big)^+ + \bar{\lambda}_t \sum_{i \neq 0} J^i_t \overline{\mathrm{IM}}^i_t - r_t \mathrm{CA}_t\Big) dt - \eta_t d\mathcal{M}_t.
\end{aligned}
\tag{22}
$$

Proof. Self-financing condition. ∎

Proposition 3

In the absence of shareholders arbitrage, in the case of a portfolio of bilateral trades between the bank and n counterparties $i = 1, \dots, n$, the CA value process satisfies the following BSDE: $\mathrm{CA}_{\bar\tau} = 0$ *and, for* $t \in (0, \bar\tau]$,

$$
\begin{aligned}
d\mathrm{CA}_t &= -\sum_{i\neq 0}(1-R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+\delta_{\tau_i}(dt) - \mathrm{UCVA}_\tau\delta_\tau(dt) + \eta_t d\mathcal{M}_t \\
&- \Big(\lambda_t\big(\sum_{i\neq 0} J^i_t P^i_t - \mathrm{EC}_t(L) - \mathrm{CA}_t\big)^+ + \bar\lambda_t \sum_{i\neq 0} J^i_t \overline{\mathrm{IM}}^i_t - r_t\mathrm{CA}_t\Big)dt + dL_t,
\end{aligned}
\tag{23}
$$

for some risk-neutral local martingale L interpreted as the CA *desk loss process. Assuming integrability, it holds that*

$$
\begin{aligned}
\mathrm{CA}_t &= \underbrace{\mathbb{E}_t \sum_{t<\tau_i} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+}_{\mathrm{UCVA}_t} + \underbrace{\mathbb{E}_t \int_t^{\bar\tau} \beta_t^{-1}\beta_s\bar\lambda_s \sum_{i\neq 0} J^i_s \overline{\mathrm{IM}}^i_s ds}_{\mathrm{MVA}_t} \\
&+ \underbrace{\mathbb{E}_t \int_t^{\bar\tau} \beta_t^{-1}\beta_s\lambda_s\Big(\sum_{i\neq 0} J^i_s P^i_s - \mathrm{EC}_s(L) - \mathrm{CA}_s\Big)^+ ds}_{\mathrm{FVA}_t},\quad 0 \le t \le \bar\tau.
\end{aligned}
\tag{24}
$$

Proof. Risk-neutral martingale condition on L+ terminal condition $\mathrm{CA}_{\bar\tau} = 0$. ∎

- Unless $\lambda = 0$, the CA and FVA BSDEs are made nonstandard by the term $\mathrm{EC}_t(L)$, which entails the conditional law of the one-year-ahead increments of the martingale component L of their solution.
  - The mathematical well-posedness of these nonstandard BSDEs is studied in Crépey and Élie (2016).
- Since EC is only affected by the time fluctuations of L, the BSDE (23) is independent of the accrued loss y, which therefore does not affect CA, nor the KVA. In conclusion, the value of the accrued loss y is immaterial.
- In the same line of thinking, one can observe that introducing the financial contracts one after the other in one or the reversed order in the portfolio at time 0 would end-up in the same aggregated FTP amounts for the bank (the ones corresponding to the portfolio CA and KVA), but, interestingly enough, in quite different FTPs for the different counterparties.

CA-CL BSDE

- In this part we revisit the above results when we remove the constraint that a bank is an intrinsically leveraged entity, as if shareholders were able to redeem all debt of the bank, so that the interest of shareholders and creditors would be aligned to each other.
- In this case, one must reckon the windfall benefits of the bank at its own default time and compute contra-assets and contra-liabilities altogether.
- We denote their aggregated value by $\mathrm{CA\text{-}CL}$ and by $\widetilde{L}$ the loss process of a related "$\mathrm{CA\text{-}CL}$ desk."
- Computations then yield to distinguish four contra-liabilities:

FTDDVA: is the first-to-default DVA;

CVA^{CL}: is the difference $(\mathrm{UCVA} - \mathrm{FTDCVA})$;

DA, MDA: the funding debt adjustment (akin to the DVA2 in Hull and White (2012)) and the margin debt adjustment, are the contra-liability counterparts of the FVA and the MVA, i.e. the values of the windfall benefits of the bank at its own default time on its variation and initial margin funding debts.

Proposition 4

In the absence of bank stakeholders arbitrage (assuming in this section that the interests of the shareholders and the creditors of a bank could be aligned to each other), in the case of a portfolio of bilateral trades between the bank and n counterparties $i = 1, \dots, n$:

(i) *Contra-assets and contra-liabilities are valued altogether as*

$$
\begin{aligned}
\mathrm{CA\text{-}CL}_t = & \underbrace{\sum_{i \neq 0} \mathbb{E}_t \left[\beta_t^{-1} \beta_{\tau_i} \mathbb{1}_{\{t < \tau_i \leq \bar{\tau}\}} (1 - R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+ \right]}_{\mathrm{FTDCVA}_t = \sum_{i \neq 0} \mathrm{FTDCVA}^i_t} \\
& - \underbrace{\sum_{i \neq 0} \mathbb{E}_t \left[\beta_t^{-1} \beta_{\tau_i} \mathbb{1}_{\{t < \tau \leq \tau_i \wedge T\}} (1 - R)(Q^i_{\tau} - \overline{\mathrm{IM}}^i_{\tau})^- \right]}_{\mathrm{FTDDVA}_t = \sum_{i \neq 0} \mathrm{FTDDVA}^i_t}, \quad 0 \leq t \leq \bar{\tau}.
\end{aligned} \tag{25}
$$

Proposition 4 (Cont'd)

(ii) The FDA, MDA and $\mathrm{CVA}^{\mathrm{CL}}$ of the bank are given, for $0 \le t \le \bar{\tau}$, as

$$\begin{aligned}
\mathrm{FDA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \lambda_s \Big(\sum_{i \neq 0} J_s^i P_s^i - \mathrm{EC}_s(\widetilde{L}) - \text{CA-CL}_s \Big)^+ ds \\
&= \mathbb{E}_t \Big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau < T\}} (1-R) \Big(\sum_{i \neq 0} J_{\tau-}^i P_{\tau-}^i - \mathrm{EC}_{\tau-}(\widetilde{L}) - \text{CA-CL}_{\tau-} \Big)^+ \Big] \\
\mathrm{MDA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \bar{\lambda}_s \Big(\sum_{i \neq 0} J_s^i \overline{\mathrm{IM}}_s^i \Big)^+ ds \\
&= \mathbb{E}_t \Big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau < T\}} (1-\bar{R}) \Big(\sum_{i \neq 0} J_{\tau-}^i \overline{\mathrm{IM}}_{\tau-}^i \Big)^+ \Big] \\
\mathrm{CVA}_t^{\mathrm{CL}} &= \mathbb{E}_t \sum_{\tau < \tau_i} \beta_t^{-1}\beta_{\tau_i} (1-R_i)(Q_{\tau_i}^i - \mathrm{IM}_{\tau_i}^i)^+ .
\end{aligned} \tag{26}$$

(iii) The FTP corresponding to a new deal is given as

$$\widetilde{\mathrm{FTP}} = \Delta\text{CA-CL} + \Delta\widetilde{\mathrm{KVA}} = \Delta\mathrm{FTDCVA} - \Delta\mathrm{FTDDVA} + \Delta\widetilde{\mathrm{KVA}}, \tag{27}$$

where FTDCVA and FTDDVA are given in (25) and where $\widetilde{\mathrm{KVA}}$ is the solution to the KVA BSDE (10) with input process $\widetilde{L}$.

Proof. In order to derive the fair value of the portfolio for the bank as a whole, one needs to consider a modified martingale accounting for the cash-flows valued by the contra-liabilities.
Exploiting cancellations that then arise, we end-up with (25). ∎

- The CA-CL formula (25) is symmetrical in that each difference $(\mathrm{FTDCVA}^i - \mathrm{FTDDVA}^i)$ corresponds to the negative of the analogous quantity considered from the point of view of the counterparty i.
- It is also consistent with the Modigliani and Miller (1958) theorem, which states that the wealth of the bank as a whole does not depend on its funding policy.
- Since the presence of collateral has a direct reducing impact on FTDCVA/DVA, this formula may give the impression that collateralization achieves a reduction in counterparty credit risk at no cost to either the bank or the clients.
- However, as we discussed before, this is not the case as the funding costs for margin and the cost of capital are material and need to be reflected into entry prices.
- The CA-CL formula (25) yields an objective, reference, but mainly indicative value for the counterparty credit risk of the portfolio, which is only the best available proxy for the overall value of counterparty credit risk to all stakeholders, ignoring the misalignement of interest of bank shareholders and creditors accounted for through the CA equation (24).

Unilateral vs. First-to-Default CVA

- Ignoring the fictitious wealth transfer $\mathrm{UCVA}_\tau \delta_\tau(dt)$ in (22), one would end-up with, instead of UCVA_t in the CA (24):

$$\mathrm{FTDCVA}_t = \sum_{i \neq 0} \mathbb{E}_t \Big[\beta_t^{-1} \beta_{\tau_i} \mathbb{1}_{\{t < \tau_i \leq \bar{\tau}\}} (1 - R_i)(Q^i_{\tau_i} - \mathrm{IM}^i_{\tau_i})^+ \Big]$$

- The regulator says that a firm capital cannot be seen increasing as a consequence of the sole deterioration of the bank credit, all else being equal. This may seem reasonable, although not entirely justified by cash flow arithmetics, as visible through the fact that one has to introduce a fictitious wealth transfer $\mathrm{UCVA}_\tau \delta_\tau(dt)$ to get UCVA instead of FTDCVA in CA.
- It is therefore questionable whether one should actually use UCVA (as of the current regulatory framework) or FTDCVA in CA (hence, through the FTP formula (6), in entry prices).

Synthesis

- CA-CL and CA represent two extreme points of view, the one of the bank as a whole under the premise that the interest of bank shareholders and creditors could be aligned to each other, where all contra-liabilities are included, and the one of the regulator, where all contra-liabilities are banned.
- Actually, the interest of the shareholders properly said would correspond to a variant of CA where UCVA is replaced by FTDCVA
  - Whereas UCVA corresponds to a regulator point of view
- In principle banks are price makers, end-clients are price takers
- Banks are service providers (financial intermediaries) that can only survive in the long run by passing to the end-clients the FTP corresponding to the point of view of the bank shareholders.
- In practice the FTP effectively charged by a bank may lie somewhere between the above two extremes, also depending on its bargaining power.

Introducing coefficients $\Lambda_{cva^{cl}}$, Λ_{dva}, Λ_{fda} and Λ_{mda} in $[0,1]$ expressing how much of each of the cash-flows giving rise to the contra-liabilities the bank shareholders would be ready to pay for, the ensuing valuation of counterparty credit risk (dubbed CCR value process) would follow, by similar arguments as the ones already developed for CA and $\mathrm{CA}\text{-}\mathrm{CL}$, as

$$
\begin{aligned}
\mathrm{CCR}_t &= \underbrace{\mathrm{FTDCVA}_t + (1-\Lambda_{cva^{cl}})\mathrm{CVA}^{\mathrm{CL}}_t}_{\mathrm{CVA}_t} - \underbrace{\Lambda_{dva}\mathrm{FTDDVA}_t}_{\mathrm{DVA}_t} \\
&\quad \underbrace{+\,\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\big(\lambda_s - \gamma_s(1-R)\Lambda_{fda}\big)\Big(\sum_{i\neq 0} J^i_s P^i_s - \mathrm{EC}_s(\widehat{L}) - \mathrm{CA}_s\Big)^+ ds}_{\text{Blended-FVA}_t} \\
&\quad \underbrace{+\,\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\big(\bar{\lambda}_s - \gamma_s(1-\bar{R})\bar{\Lambda}^{mda}\big)\sum_{i\neq 0} J^i_s \overline{\mathrm{IM}}^i_s ds}_{\text{Blended-MVA}_t},\quad 0 \le t \le \bar{\tau},
\end{aligned}
\tag{28}
$$

where $\widehat{L}$ is the loss process resulting from imperfect hedging by a “CCR desk” of all the acknowledged CCR cash-flows.

- The resulting FTP corresponding to a new deal would be given as

$$\begin{aligned}\widehat{\mathrm{FTP}} &= \Delta\mathrm{CCR} + \Delta\widehat{\mathrm{KVA}} \\ &= \Delta\mathrm{CVA} - \Delta\mathrm{DVA} + \Delta\text{Blended-FVA} + \Delta\text{Blended-MVA} + \Delta\widehat{\mathrm{KVA}},\end{aligned} \quad (2$$

where the CCR components are detailed in (28) (cf. also (25) and (26)) and where $\widehat{\mathrm{KVA}}$ is the solution to the KVA BSDE (10) with input process $\widehat{L}$.

- In view of what precedes, the point of view of the regulator (bank as a whole) would correspond to setting all the Λ coefficients to 0 (1), whereas the view of the shareholders properly said would correspond to setting all the Λ coefficients to 0, except for $\Lambda_{cva^{cl}} = 1$.

KVA Is Not a CET1 Deduction

- In Green et al. (2014) and Green and Kenyon (2016), as also discussed in some theoretical actuarial literature (see Salzmann and Wüthrich (2010, Sect. 4.4)), the KVA is treated as part of the value of the derivative portfolio (as the valuation adjustment terminology fallaciously induces to believe), as if there was a single account for reserve capital and retained earnings altogether as well as a further KVA contra-asset (see the figure).
- In reality there are two distinct balance-sheet asset lines that do not flow into each other:
  - Reserve capital, which is used for dealing with expected counterparty default losses and funding expenditures;
  - Retained earnings, which are used for remunerating the shareholders' capital at risk earmarked to absorb exceptional losses.
- In our continous-time model, the KVA in this sense would correspond to the value of an additional $-(d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt)$ or $h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$ cash-flow (up to a risk-neutral local martingale, having assumed $\mathbb{P} = \mathbb{Q}$) in the CA desk loss process (22). By so doing, released earnings are treated as trading losses, which they are not.

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Even then, in order to get from this (conceptually erroneous) premise the KVA-inclusive XVA equation of Green et al. (2014) or Green and Kenyon (2016), one needs to make two successive approximations (otherwise the resulting KVA-inclusive XVA equation is not only erroneous conceptually, but also untractable numerically):

- Ignoring that the KVA is loss-absorbing, i.e. simplifying $h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$ into $h\mathrm{EC}_t dt$.
  - Otherwise the KVA itself would appear in the modified loss process (22), making the resulting CA and FVA BSDEs intractable;
- Working with regulatory instead of economic capital.
  - Otherwise forward starting one-year-ahead fluctuations of the KVA and of exceptional losses should be simulated for capital and in turn KVA calculation. This would both involve a conceptual circularity and be intractable numerically.
  - Using regulatory instead of economic capital is motivated by practical considerations but loses self-consistency
    - Loses the connection where the KVA input should be the loss process arising from (CCR) mis-hedge.

- Starting from the wrong premise and using these approximations, one obtains a KVA-inclusive XVA equation as of (23) with one more term $h\mathrm{EC}_t(L)dt$ (corresponding to the KVA) in the right-hand side, which is basically (modulo the last point above) the KVA-inclusive XVA equation or Green et al. (2014) and Green and Kenyon (2016).
- In Green et al. (2014) and Green and Kenyon (2016), this equation is derived based on a replication argument, which runs counter to the very notion of the KVA as the cost of market incompleteness.

Starting from the wrong premise and using these approximations, one obtains a single, KVA-inclusive XVA equation, with one additional $h\mathrm{EC}_t dt$ term on top of the CVA and FVA terms.

- The ensuing KVA is "too high" because it misses the required "$+h$" in the discount rate
- This may lead to paradoxical situations where the KVA is greater than the capital it is issued from
- This is partially mitigated by the fact that this KVA is (unduly) computed with regulatory capital, which is in principle a lower bound on economic capital

However, why should one do that??

- The "right" KVA equation, on top of being well-grounded economically, does not require the distorting approximations of the previous slides
- It never leads to KVA$>$ EC.

Outline

Picard iteration

$L^{(0)} = y$, $\mathrm{CA}^{(0)} = \mathrm{CA}^\star$ and, for $k \geq 1$,

$$
\begin{aligned}
& L_0^{(k)} = y \text{ and, for } t \in (0, \bar{\tau}], \\
& dL_t^{(k)} = d\mathrm{CA}_t^{(k-1)} dt - r_t \mathrm{CA}_t^{(k-1)} dt + \sum_{i \neq 0} (1 - R_i)(Q_{\tau_i}^i - \mathrm{IM}_{\tau_i}^i)^+ \delta_{\tau_i}(dt) \\
& + \mathrm{UCVA}_\tau \delta_\tau(dt) + \lambda_t \Big(\sum_{i \neq 0} J_t^i P_t^i - \mathrm{EC}_t\big(L^{(k-1)}\big) - \mathrm{CA}_t^{(k-1)} \Big)^+ dt \\
& + \bar{\lambda}_t \sum_{i \neq 0} J_t^i \overline{\mathrm{IM}}_t^i dt - \eta_t d\mathcal{M}_t; \\
& \mathrm{CA}_t^{(k)} = \mathbb{E}_t \sum_{t < \tau_i \leq \bar{\tau}} \beta_t^{-1} \beta_{\tau_i} (1 - R_i)(Q_{\tau_i}^i - \mathrm{IM}_{\tau_i}^i)^+ + \mathbb{E}_t \big[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{t < \tau < T\}} \mathrm{UCVA}_\tau \big] \\
& + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s \lambda_s \big(\sum_{i \neq 0} J_s^i P_s^i - \mathrm{EC}_t\big(L^{(k)}\big) - \mathrm{CA}_s^{(k-1)} \big)^+ ds \\
& + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s \bar{\lambda}_s \sum_{i \neq 0} J_s^i \overline{\mathrm{IM}}_s^i ds \,, \quad t \in [0, \bar{\tau}].
\end{aligned}
\tag{30}
$$

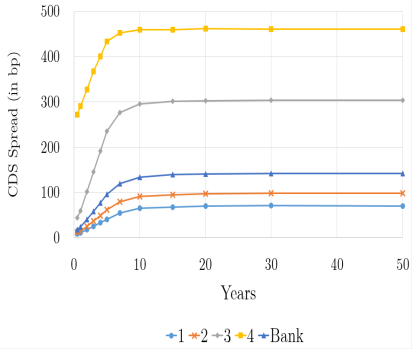
Toy Portfolio

We first consider a portfolio of ten USD currency fixed-income swaps on the date of 11 January 2016 (without initial margins, i.e. for IM $= 0$).

Toy portfolio of swaps (the nominal of each swap is $\$10^4$)

Mat.	Receiver Rate	Payer Rate	i
10y	Par 6M	LIBOR 3M	3
10y	LIBOR 3M	Par 6M	2
5y	Par 6M	LIBOR 3M	2
5y	LIBOR 3M	Par 6M	3
30y	Par 6M	LIBOR 3M	2
30y	LIBOR 3M	Par 6M	1
2y	Par 6M	LIBOR 3M	1
2y	LIBOR 3M	Par 6M	4
15y	Par 6M	LIBOR 3M	1
15y	LIBOR 3M	Par 6M	4

Credit curves of the bank and its four conterparties



Toy portfolio. *Left*: XVA values and standard relative errors (SE). *Right*: Respective impacts when Swaps 5 and 9 are added last in the portfolio.

	\$Value	SE
UCVA_0	471.23	0.46%
$\mathrm{FVA}_0^{(0)}$	73.87	1.06%
FVA_0	3.87	4.3%
KVA_0	668.83	N/A
FTDCVA_0	372.22	0.46%
FTDDVA_0	335.94	0.51%

	Swap 5	Swap 9
$\Delta\mathrm{UCVA}_0$	155.46	-27.17
$\Delta\mathrm{FVA}_0^{(0)}$	-85.28	-8.81
$\Delta\mathrm{FVA}_0$	-80.13	-5.80
$\Delta\mathrm{KVA}_0$	127.54	-52.85
$\Delta\mathrm{FTDCVA}_0$	98.49	-23.83
$\Delta\mathrm{FTDDVA}_0$	122.91	-80.13

Return

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Representative Portfolio

We now consider a representative portfolio with about 2,000 counterparties, 100,000 fixed income trades including swaps, swaptions, FX options, inflation swaps and CDS trades ($IM = 0$).

XVA	\$Value
$UCVA_0$	242 M
$FVA_0^{(0)}$	126 M
FVA_0	62 M
KVA_0	275 M
FTDCVA	194 M
FTDDVA	166 M

Return

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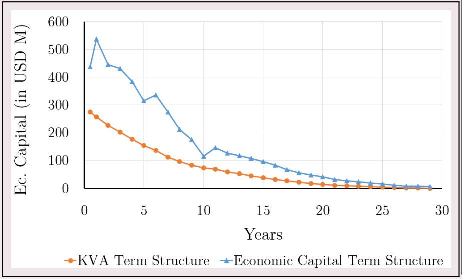
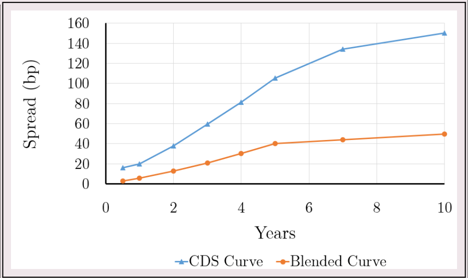
Left: Term structure of economic capital compared with the term structure of KVA.
Right: FVA blended funding curve computed from the ground up based on capital projections.





Return to FVA | Return to KVA | Return to SCR | Return to MC

Take-away Messages

- We reconsider XVAs in the light of a structural model of the bank.
- The fact thats banks are intrinsically leveraged entities invalidates several of the conclusions of Modigliani-Miller theory but not all.
- We introduce a Solvency II inspired framework for assessing KVA, understood as cost of capital, for a bank, pass it on to the bank's clients and distribute it gradually to the bank's shareholders through a dividend policy which would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future.
  - It is stipulated in IFRS 4 that "It does not apply to other assets and liabilities of an insurer, such as financial assets and financial liabilities within the scope of IFRS 9 Financial Instruments." (http://www.ifrs.org/IFRSs/Documents/Technical-summaries-2014/IFRS 4.pdf). Of course accounting boards and regulators have to limit the reach and scope of a document. We are talking in the present presentation about principles, not drafting a regulatory document.
  - This said, even under the current regulatory environment, it would be possible for a bank to propose to follow the EC, KVA and FVA schemes of this paper. Regulators would just have to look at it and review the merits of the proposal.
  - The technology that it requires is already there and affordable.

- While insurance portfolios have only a KVA-like metric called risk margin, banks have several other XVA metrics. The fungibility of capital as a source of funding for variation margin leads to a significant FVA reduction.
- Our FVA is defined asymmetrically since in no way we can recognise, even approximately, a positive funding benefit to excess capital at hand in the future.

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