To Borrow or Insure? Long Term Care Costs and the Impact of Housing

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Topic Coverage

Topic coverage

Research Motivation

- Health status and health costs a major longevity risk in retirement
- Health state important in retirement planning (Ameriks *et al.*, 2011; Yogo, 2009)
- Large component of wealth in illiquid home equity for 65 year olds in many countries
- Housing and LTC are complementary ways of financing or insuring health costs (Davidoff, 2010)
- LTC costs are increasing and more will be covered by individual co-payments (Shi and Zhang, 2013)
  - Australia: government requires co payment for LTC costs, potential use of home equity to fund costs
  - U.S.: Medicaid and Medicare + private insurance + personal payment
  - The private LTC insurance market will be an important supplement (Colombo *et al.*, 2011)

Research Questions

- To what extent should reverse mortgage and private long-term care insurance (LTCI) be used by individuals to finance or insure longevity and LTC retirement risks?
- What is the impact of house price growth and health state on retirees' optimal consumption, reverse mortgage and LTC insurance decisions?
- Should individuals borrow against a residential home or insure LTC? Does borrowing reduce the need for LTC insurance?
- To what extent does LTC insurance improve individual welfare gains allowing for housing and access to reverse mortgages?
- What computational methods are required to implement more complex individual life-cycle decision models incorporating housing and health states?

Topic coverage

Incorporation of health status and house price

- LTCI and house price dynamics in the optimal portfolio choice field
    - Not including housing asset (Ameriks *et al.*, 2011)
    - Deterministic, Binomial, Log-Normal house prices (Yogo, 2009; Davidoff, 2010)
- Health state adds to the dimension of the problem
- We use health states model estimated from HRS panel data (Fong *et al.*, 2015)
- We use a more realistic time series house price model (e.g., Chen *et al.*, 2010; Lee *et al.*, 2012; Yang, 2011)
- Path dependent house price dynamics - adds complexity in lifecycle model requiring more advanced numerical solution techniques
- Continuous and discrete state variables - endogenous grid and regression approaches.

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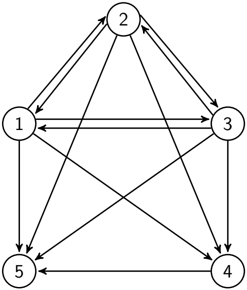
Model Structure

- Life cycle model, includes housing and reverse mortgages as well as health states
- Financial assets
  - Risk-free asset
  - House (illiquid asset, endowed wealth at retirement)
  - Reverse mortgage loan (lump sum at age 65)
  - Long-term care insurance (purchased at age 65)
- For illustration we assume $500,000 liquid wealth and $300,000 housing wealth
- Risks
  - Health dynamics and mortality risk: Markov model
  - House price: ARIMA-GARCH (more realistic, mean reversion)

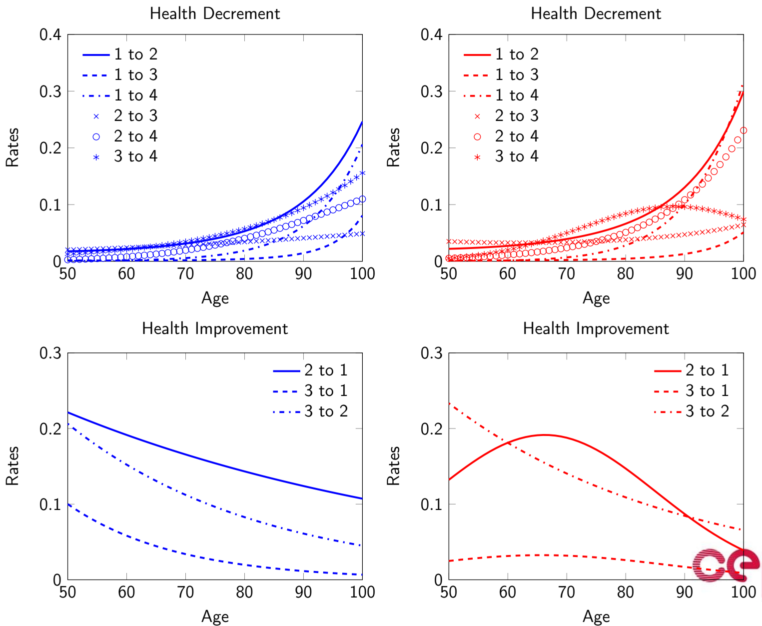
Health Dynamics

- 1 - Healthy (difficulty in no ADLs)
- 2 - Mildly disabled (difficulty in 1 ADL) and staying at home
- 3 - Severely disabled (difficulty in 2+ ADLs) and staying at home
- 4 - Institutionalized
- 5 - Dead

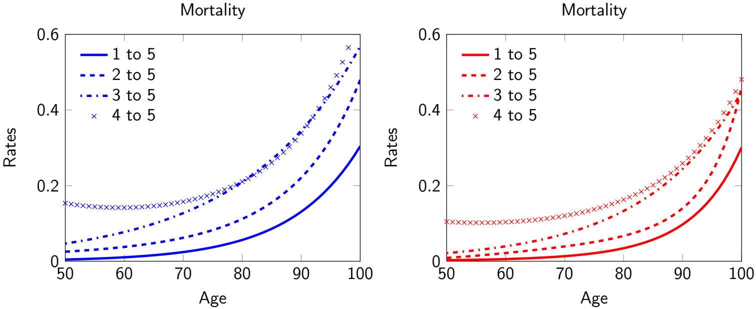


- Health transition rates/probabilities estimated using GLM (Fong *et al.*, 2015)
- Data: Health and Retirement Study (HRS) - Individual survey data for waves 1998 to 2010

Health Transition Rates



Mortality Rates



Long-Term Care Costs

- LTC costs
  - depend on health states $i \in \{2, 3, 4\}$ (\$20,000, \$40,000, \$80,000), typical based on US data
  - increase at the inflation rate f_s (assumed 1% p.a.)

$$LTC_t^i = LTC^i \exp\left(\sum_{s=1}^{t} f_s\right) \tag{1}$$

- Public LTC coverage: $GI = 10\%$ (allowing individual choice versus currently where Medicare and Medicaid "crowds out" private provision)

Private Long-Term Care Insurance

- Funds LTC costs when severely disabled (State 3) or moving to LTC facilities (State 4)
  - paying premium at age 65
  - choosing coverage $[0,\ 1-GI]$
  - actuarially fair premium calculated using estimated health dynamics: zero loadings

$$\pi_{PI,x} = \sum_{s=1}^{\infty} \sum_{j=3}^{4} p_{x:x+s}^{1j} \ LTC^{j} \ e^{(f-r_f)s} \tag{2}$$

Insurance Premium

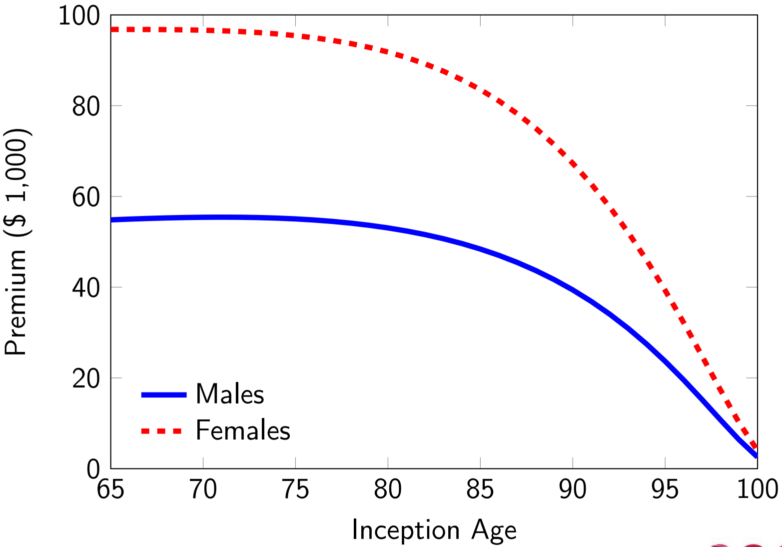


Figure. Lump sum premiums.

House Value Model

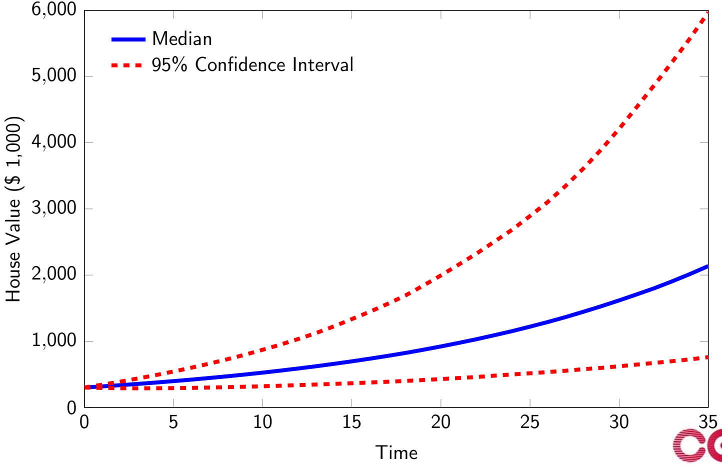
- Capital Growth: ARMA-GARCH

$$\begin{aligned} y_t &= \psi_y + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j z_{t-j} + z_t, \\ \sigma_t^2 &= \psi_{\sigma^2} + \sum_{i=1}^{m} \mu_i \sigma_{t-i}^2 + \sum_{j=1}^{n} \nu_j z_{t-j}^2, \end{aligned} \tag{3}$$

  - y_t: house price growth rate
  - σ_t^2: conditional variance given information up to $t-1$
  - We select the optimal lags in the ARMA-GARCH model (Li *et al.*, 2010; Chen *et al.*, 2010)
  - The optimal specification is a ARMA(2,4)-GARCH(1,1)

House Price Projection

Figure. House value projections based on the ARMA(2,4)-GARCH(1,1) model of house value growth rates. House value at age 65 assumed to be $300,000.



Reverse Mortgage

Reverse mortgage loan balance

$$RMLB_t = \begin{cases} RM \cdot e^{(r_f+\pi)t}, & \Lambda_t \in \{1,2,3\} \\ 0, & \Lambda_t \in \{4,5\} \end{cases} \quad (4)$$

- RM: lump sum reverse mortgage loan at age 65
- r_f: risk-free rate (assumed equal to 2% p.a.)
- π: mortgage insurance premium rate for providing no-negative equity guarantees (Shao *et al.*, 2015; Chen *et al.*, 2010) 1.5% p.a.
- Repayment is triggered when admitted to LTC facilities (State 4) or on death (State 5)

$$\min\{RMLB_t,\ HV_t\} \quad (5)$$

Topic coverage

Utility and Bequest

- Utility for both consumption and housing (imputed rent)

$$U(C_t, H_t) = \frac{\left(C_t^{\eta} H_t^{1-\eta}\right)^{1-\gamma}}{1-\gamma}, \tag{6}$$

  - C_t: non-housing consumption
  - H_t: housing consumption
  - γ: the risk aversion parameter (assumed $= 5$)
  - η: Cobb-Douglas aggregation parameter (assumed $= 0.736$, consistent with Nakajima and Telyukova (2014))
  - Constant relative risk aversion

$$R(C_t) = -C_t \frac{\partial^2 U/\partial C_t^2}{\partial U/\partial C_t} = \gamma\eta + 1 - \eta$$

Bequest motive

$$B(W_t) = \begin{cases} \frac{\beta}{1-\gamma}\left(\phi + \frac{W_t}{\beta}\right)^{1-\gamma}, & W_t \geq 0 \\ \frac{\beta}{1-\gamma}\phi^{1-\gamma}, & W_t < 0 \end{cases}, \tag{7}$$

- β: bequest motive strength (= 32.3, consistent with Ameriks *et al.* (2011))
- W_t: bequest wealth
- γ: risk aversion parameter
- CRRA type
- ϕ avoids negative infinite bequest utility, (= 7.55, consistent with Ameriks *et al.* (2011))

Topic coverage

Utility Maximization

$$V(t, i, G_t) = \max_{O_t} \mathbb{E}\left[U(C_t, H_t) + \alpha \left(\sum_{j \neq 5} p_{x+t}^{ij} V(t+1, j, G_{t+1}) + p_{x+t}^{i5} B(W_{t+1}) \right) \Big| \mathcal{F}_t \right]$$

s.t. Wealth Dynamics

- $O_t = (C_t,\ RM,\ PI)$: choice variables
- i: health state
- $G_t = (B_t,\ HV_{1:t})$: non-health state variables
- p_{x+t}^{ij}: annual probability of transitions from State i to State j
- $V(t, i, G_t)$: value function (note $\alpha = 0.96$)
- Optimization methods:
  - **Endogenous Grid Method** to avoid time-consuming root-finding routine
  - **Regression method** to allow for path dependent house price dynamics and avoid the "Curse of Dimensionality"

Liquid Wealth - At Retirement

$$
B_1 = \begin{cases} e^{r_f}\left[B_0 - C_0 + RM - \pi_{PI,x} PI\right], & i = 1 \\ e^{r_f}\left[B_0 - C_0 + RM - (1 - GI)\cdot LTC_0^i\right], & i = 2 \\ e^{r_f}\left[B_0 - C_0 + RM - (1 - GI)\cdot LTC_0^i\right], & i = 3 \\ e^{r_f}\left[B_0 - C_0 - (1 - GI)\cdot LTC_0^i\right], & i = 4 \end{cases}
$$

$$
\begin{aligned}
0 \;&\leq\; RM \;\leq\; HV_0 \cdot MLTV_x, \\
0 \;&\leq\; PI \;\leq\; 1 - GI,
\end{aligned}
$$

- B_t: liquid wealth
- RM: reverse mortgage loan, capped based on maximum loan-to-value ratio
- PI: percentage cover of private long-term care insurance
- GI: government funded coverage
- LTC_t^i: LTC costs in State i

Liquid Wealth - After Retirement

For $1 \leq t \leq T-1$

$$
B_{t+1} = \begin{cases} e^{r_f}\Big[B_t - C_t\Big], & i=1 \\ e^{r_f}\Big[B_t - C_t - (1-GI)\cdot LTC_t^i\Big], & i=2 \\ e^{r_f}\Big[B_t - C_t - (1-GI-PI)\cdot LTC_t^i\Big], & i=3 \\ e^{r_f}\Big[B_t - C_t + \mathbb{1}_{\{\Lambda_{t-1}\in\{1,2,3\}\}}\cdot \max\Big\{HV_t - RM\cdot e^{(r_f+\pi)t},\ 0\Big\} & \\ \quad -(1-GI-PI)\cdot LTC_t^i\Big], & i=4 \end{cases}
$$

$$
B_t \geq \underline{B}
$$

Bequest Wealth

$$W_{t+1} = \begin{cases} B_{t+1} + \max\left\{HV_{t+1} - RM \cdot e^{(r_f+\pi)(t+1)},\ 0\right\}, & i \in \{1,2,3\} \\ B_{t+1}, & i = 4 \end{cases}$$

- Bequest wealth: liquid wealth plus net home equity, if any (home equity depends on health state and RM choice)

Optimization: EGM

First-order condition:

$$C_t = \left\{ \frac{\alpha H_t^{\nu}}{\eta} E_t \left[e^{r_f} \left(\sum_{j \neq 5} p_{x+t}^{ij} V_B'(t+1, j) + p_{x+t}^{i5} \beta W_{t+1}^{-\gamma} \right) \right] \right\}^{\frac{1}{\nu-\gamma}}$$

Envelope condition:

$$V_B'(t, i) = \eta C_t^{\nu-\gamma} H_t^{-\nu}$$

- C_t on both sides of equation
- Conventional Exogenous Grid Method requires a time-consuming root-finding routine
- **Endogenous Grid Method** avoids need for root-finding
- Main idea is to use after-consumption wealth as grid

Optimization: Regression Approach

- House price is path dependent: ARIMA(2,1,4)-GARCH(1,1)
- Including past and current house values as separate dimensions in the grids: Curse of Dimensionality!
- **Regression method**: allow for path dependent house price dynamics and avoid the "Curse of Dimensionality"
- Simulate house value paths and regress **conditional expectations** in first-order conditions with respect to past and current house values

Optimization: Simulation

- Simulate N house value sample paths: $HV_{1:t}^{(l)}$
- At each grid point $g \in \mathcal{G}$, health state $i \in \mathcal{G}^{\Lambda}$, and **each house value path**

$$\begin{aligned} \varepsilon_t(g, i, HV_{1:t}^{(l)}) &= E_t\left[\eta\left(g, i, HV_{1:t}^{(l)}, HV_{t+1}\right) \middle| \mathcal{F}_t\right] \\ \tilde{O}_t(g, i, HV_{1:t}^{(l)}) &= \zeta\left(\varepsilon_t(g, i, HV_{1:t}^{(l)})\right) \end{aligned}$$

- $\varepsilon_t(g, i, HV_{1:t}^{(l)})$: conditional expectation function
- $\eta(\cdot)$: function to be estimated with regression within the conditional expectation operator
- $\tilde{O}_t(g, i, HV_{1:t}^{(l)})$: optimal choice

Optimization: Regression

$$
\begin{aligned}
&\eta\left(g, i, HV_{1:t}^{(l)}, HV_{t+1}\right) \\
= \quad & a(g,i) + \sum_{k=0}^{K}\sum_{d=1}^{D} b_{kd}(g,i)\left(HV_{t-k}^{(l)}\right)^{d} + \sum_{d=1}^{D} c_d(g,i) O_t^d + e_l,
\end{aligned}
$$

- K: lag length of past house values that have impact on the optimal portfolio choice
- D: the polynomial order of house values and choice variables
- $a(g,i)$, $b_{kd}(g,i)$ and $c_d(g,i)$ parameters to be estimated
- e_l: random error that corresponds to uncertainties in HV_{t+1}
- R^2 very high - around 0.99

Optimization: Approximation

- Conditional expectation:

$$\begin{aligned} & \hat{\varepsilon}_t(g, i, HV_{t-K:t}) \\ = \quad & \hat{a}(g,i) + \sum_{k=0}^{K} \sum_{d=1}^{D} \hat{b}_{kd}(g,i) \left(HV_{t-k}\right)^d + \sum_{d=1}^{D} \hat{c}_d(g,i) O_t^d \end{aligned}$$

- Approximate optimal choice:

$$\hat{O}_t(g, i, HV_{1:t}) \quad = \quad \zeta\left(\hat{\varepsilon}_t\left(g, i, HV_{t-K:t}, O_t\right)\right)$$

Parameters for base case

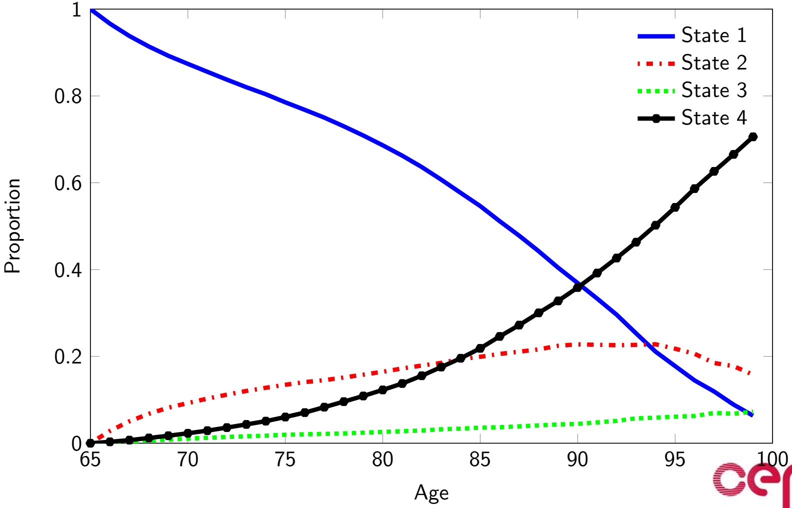
Based on prior studies (e.g, Yogo, 2009; Ameriks *et al.*, 2011; Nakajima and Telyukova, 2014).

Parameter	Explanation	Value	Source
r_f	Risk-free rate	2.00%	U.S.Treasury data
f_t	Inflation rate	1.00%	Assumption
γ	Risk aversion	5	All 3 studies
α	Utility discount factor	0.96	All 3 studies
η	Non-housing consumption aggregation	0.736	Nakajima and Telyukova (2014)
β	Bequest motive strength	32.30	Ameriks *et al.* (2011)
ϕ	Degree of bequest as luxury goods	7.55	Ameriks *et al.* (2011)
W_0	Initial wealth	\$200k	Assumption
H_0	Initial house value	\$300k	Assumption
$\underline{C}$	Consumption floor	\$4,630	Ameriks *et al.* (2011)
h_1	Housing consumption as a proportion of house value if living in the house	5%	Assumption
h_2	Housing consumption as a proportion of house value if living in nursing homes	2.5%	Assumption
LTC^2	Initial annual LTC cost in State 2	\$20k	Genworth data
LTC^3	Initial annual LTC cost in State 3	\$40k	Genworth data
LTC^4	Initial annual LTC cost in State 4	\$80k	Genworth data

Topic coverage

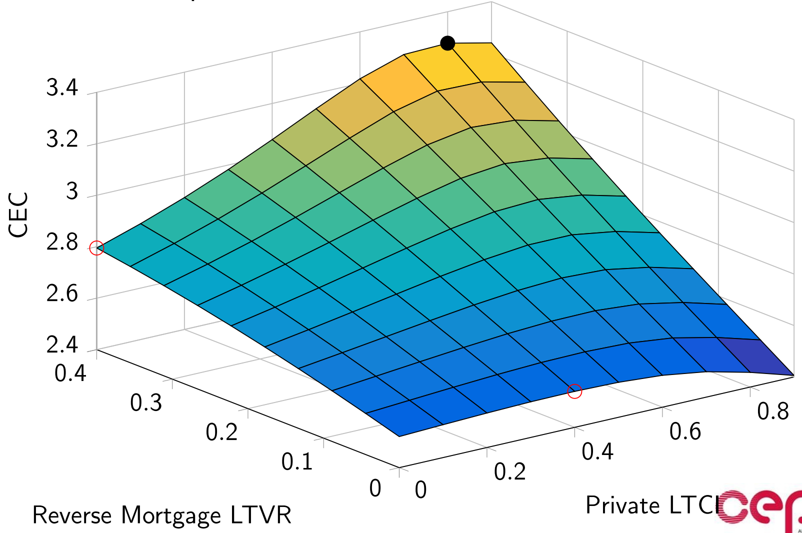
Proportion of the Alive

For initial cohort of 100,000 65-year-old healthy females: older age results driven by number in State 4



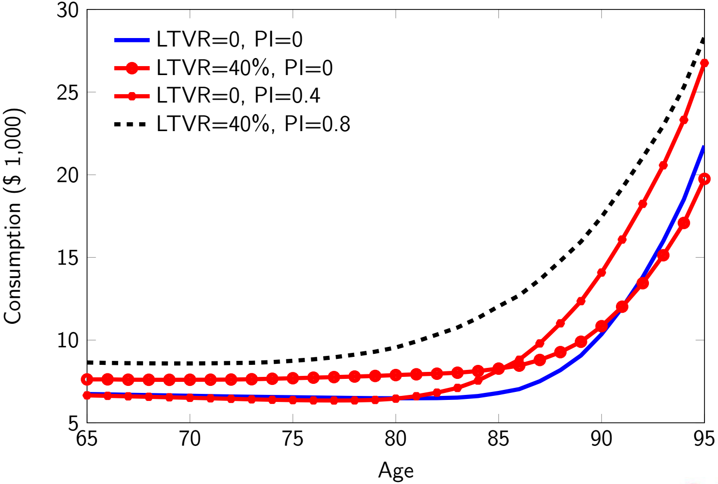
Optimal RM and Private LTCI

Certainty Equivalent Consumption (CEC) for a 65-year-old female endowed with $500k initial liquid wealth and a house worth $300k.



Optimal Consumption Path

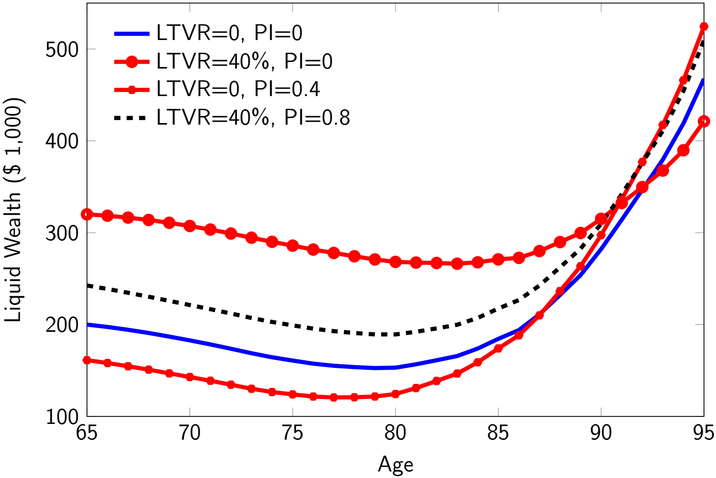
House price growth dominates older ages



- RM increases earlier age consumption and reduces older age consumption
- LTC reduces earlier age consumption and increases consumption at older ages

Optimal Liquid Wealth Path

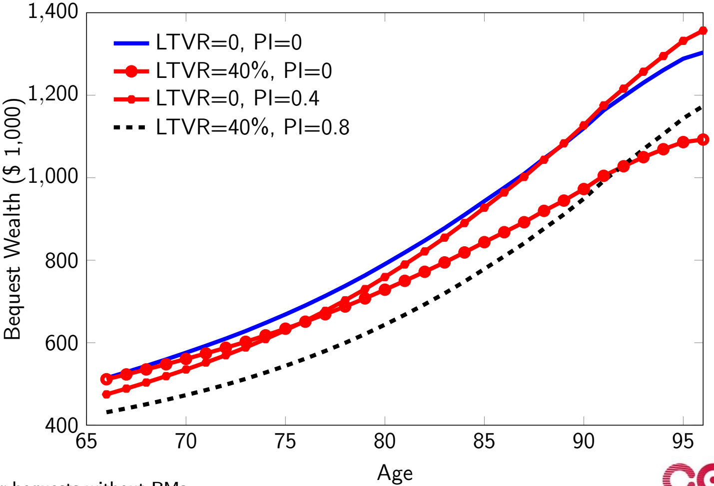
House value released into liquid wealth at older ages when moving into care



- Lump sum RM increases liquid wealth at earlier ages
- LTC lowers liquid wealth at earlier ages

Optimal Bequest Wealth Path

House price growth drives older age bequests, differing between LTC and RM's



- Higher bequests without RMs
- Lower bequests at earlier ages due to single premiums for LTC, but higher older age bequest than RM's

Welfare Analysis

Table. Percentage increase of the value function achieved when retirees have access to reverse mortgage loans and/or long-term care insurance.

	No Private LTCI	With Private LTCI
No Reverse Mortgage	0	2.18%
With Reverse Mortgage	26.21%	53.84%

Table. Retirees' willingness to pay (in $1,000) for having access to reverse mortgage loan and/or long-term care insurance.

Reverse Mortgage	Private LTCI	Reverse Mortgage and Private LTCI
66.09	4.74	175.85

- Home equity and access to RMs dominate LTCI
- Optimal to include both reverse mortgage and private LTCI

Robustness of Results - Wealth Levels and Housing

Table. Optimal reverse mortgage loan-to-value ratio (LTVR) and private LTC insurance cover (PI) for different wealth endowments (in $1,000).

Scenario	Wealth			Only LTVR	Only PI	Both	
	Total	Liquid	Housing			(LTVR	PI)
Scen 1.1	500	20%	80%	0.4	0	0.4	0.7
Base	500	40%	60%	0.4	0.4	0.4	0.8
Scen 1.2	500	80%	20%	0.1	0.8	0.4	0.9
Scen 2.1	200	20%	80%	0.4	0	0.4	0
Scen 2.2	200	40%	60%	0.4	0	0.4	0
Scen 2.3	200	80%	20%	0.4	0.9	0.4	0.9
Scen 3.1	1,000	20%	80%	0.4	0	0.4	0.6
Scen 3.2	1,000	40%	60%	0.4	0.4	0.4	0.7
Scen 3.3	1,000	80%	20%	0	0.8	0.4	0.9

- Always optimal to use RMs (time preference, risk aversion in inter-temporal marginal rate of substitution)
- Housing crowds out LTC insurance

Robustness to Parameters

Scenario	Only LTVR	Only PI	Both (LTVR	PI)
Base	0.4	0.4	0.4	0.8
Risk aversion parameter				
$\gamma = 2$	0.4	0	0.4	0.8
$\gamma = 10$	0.4	0.6	0.4	0.8
Utility discount factor				
$\alpha = 0.93$	0.4	0.2	0.4	0.7
$\alpha = 0.99$	0.4	0.5	0.4	0.8
Bequest motive strength				
$\beta = 20$	0.4	0.3	0.4	0.7
$\beta = 50$	0.4	0.5	0.4	0.8
Degree of bequest as luxury goods				
$\phi = 6$	0.4	0.6	0.4	0.8
$\phi = 9$	0.4	0.2	0.4	0.7
Housing consumption if living in the house				
$h_1 = 2.5\%$	0.4	0	0.4	0.7
$h_1 = 10\%$	0.4	0.7	0.4	0.8
Housing consumption if living in a nursing home				
$h_2 = 1\%$	0.4	0.7	0.4	0.8
$h_2 = 5\%$	0.4	0	0.4	0.7
Initial annual LTC expenses: $[LTC^2, LTC^3, LTC^4]$				
$[10k 20k 40k]	0.4	0.2	0.4	0.7
$[50k 100k 200k]	0.4	0.1	0.4	0.8

Optimal reverse mortgage loan-to-value ratio (LTVR) and private LTC insurance cover (PI) for different parameter values. Initial wealth endowment is $200k liquid wealth and $300k housing wealth.

Topic coverage

Contributions

- We extend current analysis of LTC insurance and reverse mortgages using a discrete time life-cycle model, taking into account health states and house price risk
- We implement advanced numerical optimization methods to compute solutions, not previously done for this LTC analysis
- We determine the optimal portfolio choice with respect to consumption, reverse mortgage, and private long-term care insurance varies with health state in a more complex and realistic model setting
- We quantify the welfare gains from having access to both reverse mortgage and private LTCI.

Conclusions

- Borrowing against home equity dominates LTC because of higher earlier consumption in the life cycle model, also provides some longevity insurance
- LTC with lump sum premium transfers wealth from good health states to bad health states but reduces earlier consumption
- Home equity complements LTC in funding long term care costs and can "crowd out" LTC insurance
- Highest welfare benefits come from a combination of both RM's and LTC insurance, so potential for new products that combine longevity and LTC insurance (health care annuities)
- Life cycle framework does not reflect actual individual behavior, where there is limited reverse mortgage usage and limited purchase of LTC insurance: extension to more complex preferences and inclusion of market loadings.

Announcement: AFIR-ERM Colloquium in Panama City 20 to 24 August, 2017

- Of interest to finance, investment and risk management researchers and practitioners in actuarial science and finance.
- Joint with ASTIN
- Both members of AFIR-ERM section and non-members who work or research in financial risk and risk management.
- Educational workshops for South American actuaries on ERM, Derivatives Pricing, Interest rate models and Longevity risk.
- Website: http://www.actuaries.org/panama2017/ for Call for Papers.

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