An empirical analysis of unspanned risk for the U.S yield curve

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Affine term structure models

Traditionally no-arbitrage affine term structure models assume that:

- Yield curve is sufficiently describe by three latent yield factors Litterman and Scheinkman (1991) and Ang and Piazzesi (2003) and Diebold and Li (2006).
- More recently, literature highlights the importance of additional factors:
  - The first five principal components of Treasury yields are needed to explain Treasury returns Adrian et al. (2013).
  - Macro factors Cochrane and Piazzesi (2005), Cochrane and Piazzesi (2008), and Duffee (2011).
- In macro-finance affine term structure models, the macro factors that determine bond prices are fully spanned by the current yield curve.

Unspanned factors in macro-finance term structure models

- Joslin et al. (2014) argue that macro-spanning condition implies strong and often counterfactual restrictions
- Macro variables are uninformative about:
  - expected excess returns (risk premiums) and
  - future values of themselves
- New branch of affine term structure models where macro variables have an effect on bond risk premia, but do not span the cross-section of yields (Duffee (2011), Boos (2011) and Joslin et al. (2014)).

Predictors for the U.S. bond risk premia

- n-year spread of the n-year forward rate and the one-year yield Fama and Bliss (1987).
- Treasury yield spreads Campbell and Shiller (1991).
- Linear combination of five forward rates (tent-shaped factor) Cochrane and Piazzesi (2005).
- Macro factors Ludvigson and Ng (2009) and Cooper and Priestley (2009).
- On-the-run bond liquidity premium (Funding liquidity) predicts Treasury market returns Fontaine and Garcia (2011).
- Pflueger and Viceira (2013) and Gomez(2015) provide empirical evidence for TIPS liquidity as a source of predictability for U.S. Treasury bond and/or U.S. Treasury Inflation-protected bonds (TIPS).

However,

Macro factors (real output and inflation) have been usually proposed as unspanned factors.

Little attention has been paid to financial market variables as possible (un)spanned factors

This paper

This paper aims at answering if the TIPS liquidity premium is (linearly) spanned or not by the U.S yield curve.

I contribute to the existing literature by empirically testing:

1. the plausibility of the TIPS liquidity premium as an unspanned factor by using a traditional Gaussian ATSM.
2. the empirical relationship between movements in the level, slope and curvature of the U.S. yield curve and TIPS liquidity premium shocks.

In my empirical analysis

- I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk.

- I identify the liquidity component in TIPS yields through the difference between observed break-even inflation rate (BEI) and inflation swap rate (Christensen and Gillan (2011))

$$\Delta_{n,t} = IS_{n,t} - BEI_{n,t},$$

- Estimation is performed using the three-step linear regression procedure introduced by Adrian et al. (2013) and adapted by Abrahams et al. (2013) to estimate the joint pricing of TIPS and Treasury bonds.

Main results

- Liquidity premium helps to forecast U.S. bond risk premia, but it is not linearly spanned by the information in the joint yield curve.
- I show that the liquidity factor does not affect the dynamic of bonds under the pricing measure, but does affect them under the historical measure.
- The variation in the TIPS liquidity premium predicts the future evolution of the traditional yield curve factors.

1. Motivation
2. Empirical analysis
   - Testing empirical plausibility of liquidity as unspaned factor
   - Does the variation in liquidity influence the shape of the yield curve?
3. Concluding remarks

Data

- Daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak, Sack and Wright (2007, 2010).
- One-year Treasury bill from the Federal Reserve Board statistical releases.
- I compute traditional spanned factors as the first four principal components of yields.
- For liquidity, I compute the market-based measure proposed by Christensen and Gillan (2011).
- Sample period is from January 2004 to December 2012.

A. Testing Joslin's unspanning conditions

The plausibility of the TIPS liquidity premium as an unspanned factor would be defined by three empirical facts:

1. The liquidity factor has predictive power for excess returns in the U.S bond market.
2. TIPS relative liquidity premium is not linearly spanned by the information in the joint yield curve
3. The liquidity factor does not affect the dynamics of bonds under the pricing measure ($\mathbb{Q}$-measure), although does affect them under the historical measure ($\mathbb{P}$-measure).

1) Predictability results (Gomez (2015)).

- Liquidity premium for different maturities are a significant and economically relevant source of predictability for government bonds excess returns.
- Economically, an increase in liquidity premium leads to higher TIPS excess returns, but lower Nominal Treasury excess returns.

	Nominal Treasury		TIPS	
	Short-term	Long-term	Short-term	Long-term
10-year liquidity	-40bps	-282bps	90bps	-

- Statistically, liquidity premium increase the Adj-R^2 and also the R^2_{OS} for Nominal and TIPS one-year excess returns.

	Traditional factors		Including liquidity	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Nominal	2% - 20%	-0.1% - 0.3%	21% - 52%	-0.6% - 0.6%
TIPS	0.1% - 7%	-3% - 0.5%	6% - 36%	-0.7% - 0.6%

2) Projection of liquidity onto the principal components of yields on U.S. Treasury nominal and TIPS yields.

$$\Delta_{10,t} = c + b_1 PC_{1,t} + b_2 PC_{2,t} + b_3 PC_{3,t} + e_t,$$

	Coefficient	t-stat	AdjR^2
Nominal factors			
PC_1^N	-0,161	-0,250	0,145
PC_2^N	0,246	0,101	
PC_3^N	57,869	2,169	
Real factors			
PC_1^R	2,237	3,746	0,421
PC_2^R	-11,856	-4,316	
PC_3^R	54,981	3,585	

86% adn 58% of the variation in TIPS liquidity still arises from risks distinct from the yield factors

$$BEI_{n,t} = c + b_1 PC_{1,t} + b_2 PC_{2,t} + b_3 PC_{3,t} + b_4 \Delta_{10,t} + e_t,$$

A. Individual factors

	Const	PC_1	PC_2	PC_3	Liquidity	Adj R^2
Nominal factors						
Coef	2,34	0,05	-0,02	-1,45		0,31
t-stat	33,88	3,06	-0,36	-2,31		
Real factors						
Coef	2,34	-0,02	0,32	-2,11		0,45
t-stat	35,42	-0,96	4,00	-4,18		
Liquidity factor						
Coef	2,84				-0,02	0,62
t-stat	49,56				-7,76	

B. Combined factors

	Const	PC_1	PC_2	PC_3	Liquidity	Adj R^2
Nominal factors + Liquidity						
Coef	2,81	0,05	-0,02	-0,42	-0,02	0,77
t-stat	58,66	5,37	-0,65	-1,71	-10,13	
Real factors + Liquidity						
Coef	2,79	0,02	0,12	-1,17	-0,02	0,73
t-stat	70,57	1,88	2,60	-3,51	-11,88	

Liquidity factor is important for explaining the variations in Treasury and TIPS yields.

3) Gaussian ATSM.

Assume that a liquid riskless nominal zero-coupon bond price at time t with maturity n, is given by

$$P^N_{t,(n)} = \mathbb{E}^{\mathbb{Q}}_t \left[\exp \left(- \sum_{i=0}^{n-1} r^N_{t+i} \right) \right] = \exp(A^N_n + \boldsymbol{B}'^N_n \boldsymbol{X_t}), \qquad (1)$$

Similarly, the price at time t of a inflation-linked zero coupon bond that matures at time n is equal to

$$P^R_{t,(n)} = \mathbb{E}^{\mathbb{Q}}_t \left[\exp \left(- \sum_{i=0}^{n-1} r^R_{t+i} \right) \right] = \exp(A^R_n + \boldsymbol{B}'^R_n \boldsymbol{X_t}), \qquad (2)$$

where $\mathbb{E}^{\mathbb{Q}}_t$ denotes the expected value at time t under the risk-neutral measure $\mathbb{Q}$, r^N_t is the nominal risk free interest rate, and r^R_t is the real interest rate.

I assume that the dynamics of the $K \times 1$ vector of state variables $\boldsymbol{X_t}$, under the historical measure $\mathbb{P}$, is given by

$$\boldsymbol{X_{t+1}} = \boldsymbol{\Theta_1} + \boldsymbol{\Theta_2} \boldsymbol{X_t} + \boldsymbol{\nu_{t+1}}, \tag{3}$$

where $\boldsymbol{\Theta_1}$ is a $K \times 1$ vector, $\boldsymbol{\Theta_2}$ is a $K \times K$ matrix, and $\boldsymbol{\nu_t}$ is $K \times 1$ a vector which is assumed iid Gaussian with mean $\mathbb{E}_t^{\mathbb{P}}[\boldsymbol{\nu_{t+1}}] = \mathbf{0}$ and variance $\mathbb{V}_t^{\mathbb{P}}[\boldsymbol{\nu_{t+1}}] = \boldsymbol{\Sigma}$.

Unspanned liquidity factor

I introduce a partition of the factor vector $\boldsymbol{X_t}$ into spanned factors $\boldsymbol{X_t^s}$ with nonzero risk exposures, and unspanned factor l_t which has zero risk exposure

$$\begin{bmatrix} \boldsymbol{X_{t+1}^s} \\ l_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta}_1^s \\ \theta_1^l \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Theta}_2^{ss} & \boldsymbol{\Theta}_2^{sl} \\ \boldsymbol{\Theta}_2^{ls} & \theta_2^{ll} \end{bmatrix} \begin{bmatrix} \boldsymbol{X_t^s} \\ l_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu_{t+1}^s} \\ \nu_{t+1}^l \end{bmatrix}$$

where $\boldsymbol{X_t^s}$ is a $K_s \times 1$ vector such that $\boldsymbol{X_t}$ is of dimension $K \times 1$ with $K = K_s + 1$. $\boldsymbol{\Theta}_2^{ss}$ is the upper $K_s \times K_s$ matrix, and $\boldsymbol{\Theta}_2^{sl}$ and $\boldsymbol{\Theta'}_2^{ls}$ are $K_s \times 1$ vectors.

3.1) Testing if risk factor exposures associated with individual pricing factors are different from zero.

$$\log P_{t,n} = A_n + \boldsymbol{B}'_n \boldsymbol{X_t}.$$

Let $\boldsymbol{b}_i$ a particular column of $\boldsymbol{B}'$, then $Ho : \boldsymbol{b}_i = \mathbf{0}_{N\times 1}$

Factors ($\boldsymbol{X_t}$)	Nominal ($\boldsymbol{B}^N$)	TIPS ($\boldsymbol{B}^R$)	Both ($\boldsymbol{B}^N$,$\boldsymbol{B}^R$)
PC_1^N	43.13	21.72	26.03
PC_2^N	42.41	19.74	23.99
PC_3^N	14.09	20.41	34.51
PC_1^{OR}	20.24	40.01	60.25
Δ_{10}	11.13	11.23	12.35
Critical value $\chi^2_{(N,\alpha=0.05)}$	$\chi^2_9 = 16.91$	$\chi^2_8 = 15.50$	$\chi^2_{17} = 27.58$

This justify the assumption of treating the liquidity premium factor as unspanned in the specification

3.2) Assessing whether or not a given risk factor is priced in the cross-section of Treasury and TIPS returns.

Table: Market prices of risk (Λ): unspanned specification

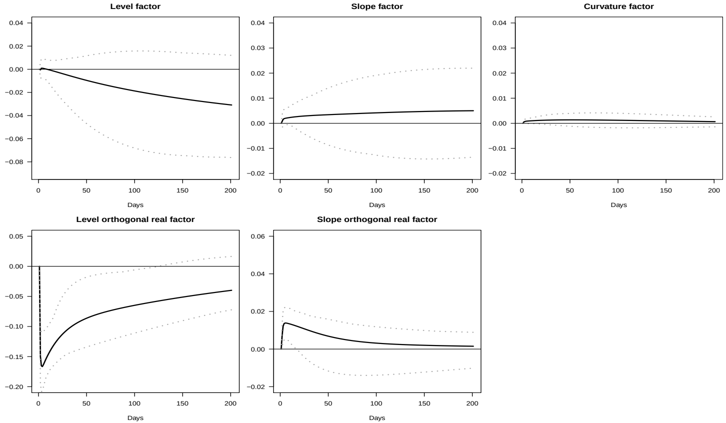
Factor	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	$\lambda_{1.5}$	$H0 : \lambda = 0$
PC_1^N	0.2313	**0.0607**	-0.1221	-0.1863	0.0055	-0.0010	37.75
	(0.3175)	(0.0231)	(0.1167)	(0.4701)	(0.2283)	(0.1112)	
PC_2^N	**0.0621**	0.0210	**-0.0427**	**-0.0827**	0.0013	-0.0003	13.50
	(0.0367)	(0.0271)	(0.0135)	(0.0543)	(0.0264)	(0.0129)	
PC_3^N	-0.0087	**-0.0022**	**0.0070**	**0.0144**	-0.0003	**0.0003**	10.01
	(0.0090)	(0.0010)	(0.0003)	(0.0013)	(0.0006)	(0.0001)	
PC_1^{OR}	-0.1728	-0.0185	**-0.1359**	-0.3650	0.0156	-0.0041	12.97
	(0.3304)	(0.0241)	(0.1125)	(0.4891)	(0.2375)	(0.1157)	

$\chi_6^2 = 12.59$ **for a significance level of** $\alpha = 5\%$

The information contained in the yield curve is insufficient to completely characterized the variation in the price of curvature risk.

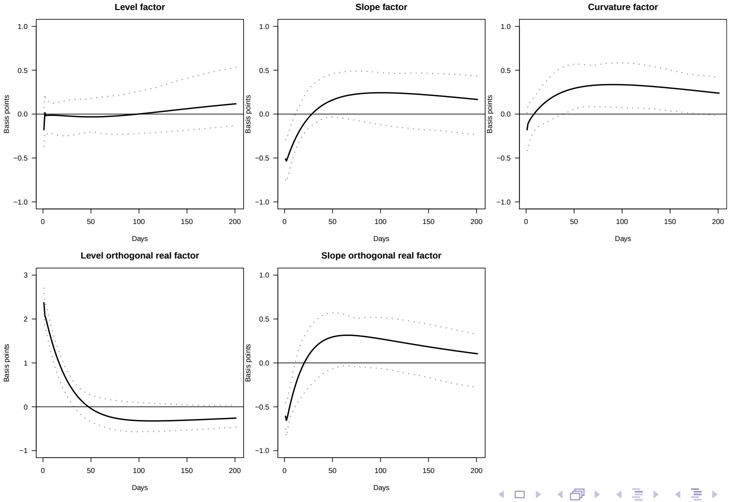
B. Does the variation in liquidity influence the shape of the yield curve?

Figure: Response of nominal and orthogonal real factors to liquidity shocks



Does the variation in liquidity influence the shape of the yield curve?

Figure: Response of liquidity to yield factor's shocks



Final Remarks

Research agenda:

- Do macroeconomic variables and bond returns impact liquidity premium?
- Does liquidity premium provide information about future macroeconomic variables?
- More general famework (evidence of mispricing in bond market).
- Improving our understanding of TIPS could potentially help to employ these securities more efficiently both from a policy but also from an investors perspective.