An Application of Extreme Value Theory for Measuring Financial Risk in the Uruguayan Pension Fund

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Introduction

- Traditional statistical methods for financial risk measures fits models to all data even if primary focus is on extremes.
- In literatura → **normal** assumption for financial returns.
- This assumption provides a good approximation for the **average** of financial returns but does not provide a good fit for the **tail** of the distribution.
- The last 20 years → instabilities in financial markets → **volatility**.
- Search for more appropriate methodologies→ **Extreme Value Theory**.

Risk Measures

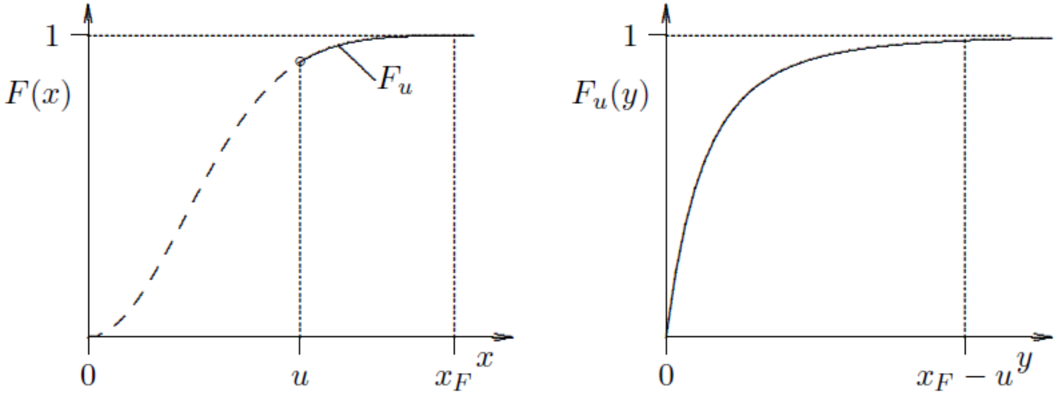
Suppose a random variable X with continuous distribution function F models losses on a certain financial porfolio over a certain time horizon.

- **Value at Risk:** $VaR_\alpha = F^{-1}(1-\alpha)$
- **Expected Shortfall**: $ES_\alpha = E(X|\, X > VaR_\alpha)$
- **Return Level**: $R_k^m = H^{-1}(1-1/m)$

If H is the distribution of the maximum observed over a successive non overlapping periods of equal length. For example, assuming a model for the annual maximum, the 15-years return level R_{365}^{15} is on average only exceded in one year out of every 15 years.

Extreme Value Theory

The Peak Over Threshold (POT) method, considers the distribution of exceedances over a certain threshold.



The distribution function F_u is called the excess distribution function and is:

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F(u+y) - F(u)}{1 - F(u)}$$

Extreme Value Theory

Theorem Pickands (1975), Balkema and de Haan (1974).

For a large class of underlying distribution F, the excess distribution function F_u can be approximated by GPD for increasing threshold u.

$$F_u(y) \approx G_{\xi,\beta}(y), \qquad u \longrightarrow \infty$$

where $G_{\xi,\beta}$ is the Generalized Pareto Distribution (GPD) which is given by

$$G_{\xi,\beta}(y) = \begin{cases} (1 + \xi y/\beta)^{-1/\xi} & \xi \neq 0 \\ 1 - e^{-y/\beta} & \xi = 0 \end{cases}$$

for $y \in [0, (x_F - u)]$ if $\xi \geq 0$ and $y \in [0, -\beta/\xi]$ if $\xi < 0$.

Risk Measures under Extreme Value Theory

Assuming a GPD function for the tail distribution, if we denote $x = u + y$ then

$$F(x) = \big(1 - F(u)\big)F_u(y) + F(u)$$

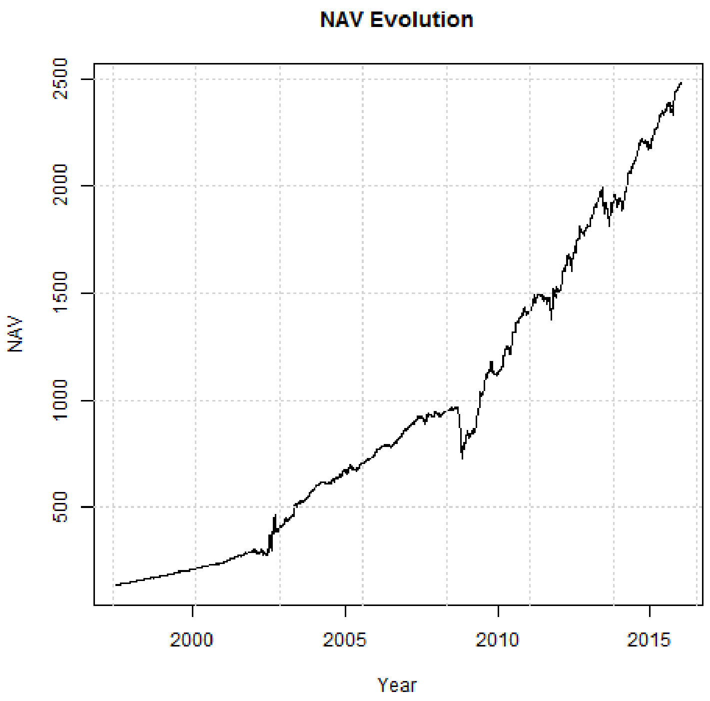
and replacing F_u by the GPD and $F(u)$ by the empiric estimate $(n - N_u)/n$, where n is the total number of observations and N_u the number of observations above the threshold u.

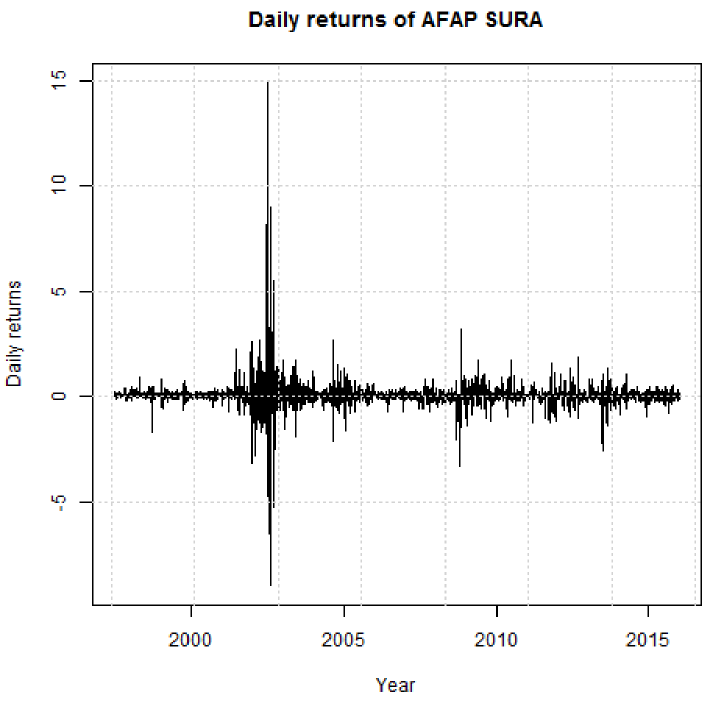
$$VaR_\alpha = u + \frac{\beta}{\xi}\left(\left(\frac{n}{N_u}\alpha\right)^{-\xi} - 1\right)$$

$$ES_\alpha = \frac{VaR_\alpha}{1-\xi} + \frac{\beta - \xi u}{1-\xi}$$

Empirical Results

We work with the daily losses series of AFAP SURA NAV over a period of 18 years (1997-2015). Containing 4.802 trading days.



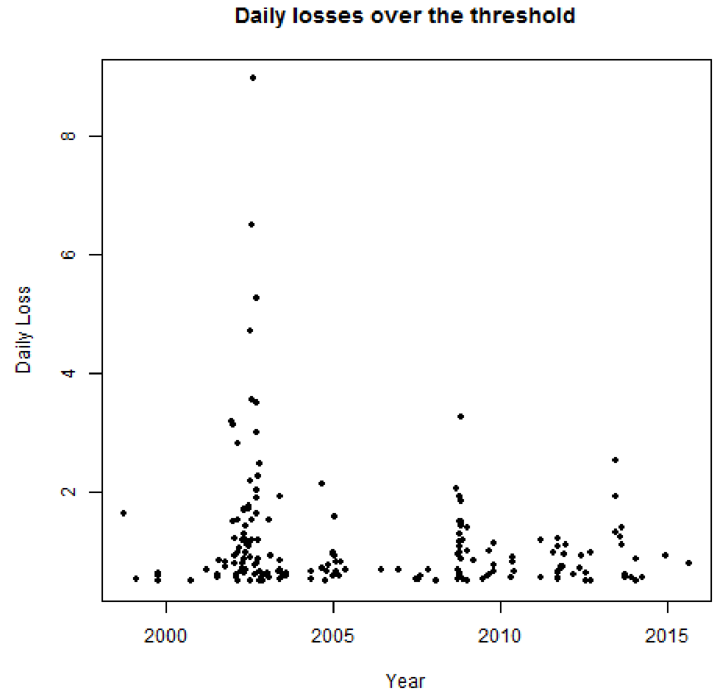


Don’t appear to have a normal distribution

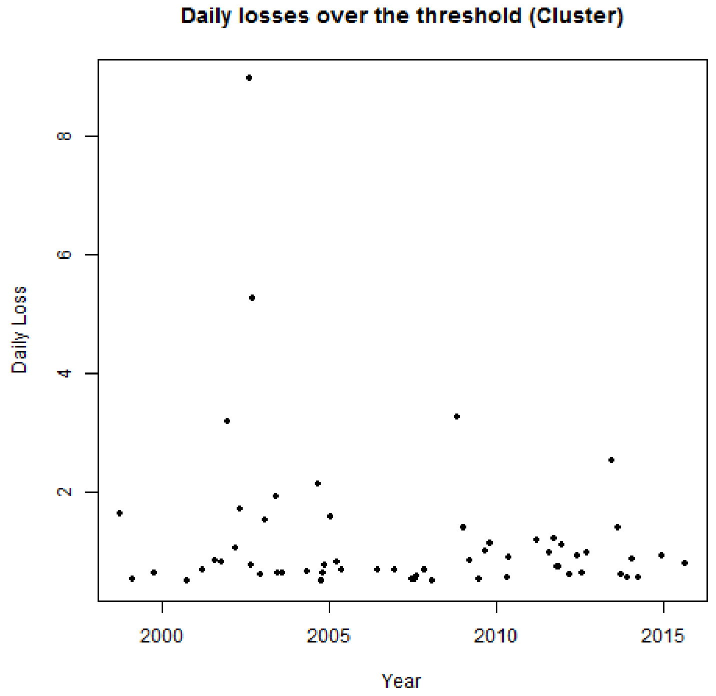
Empirical Results

In practice, we have to consider two important aspects, the selection of the threshold u and the Independence of the exceedances.

$u = 0,5$



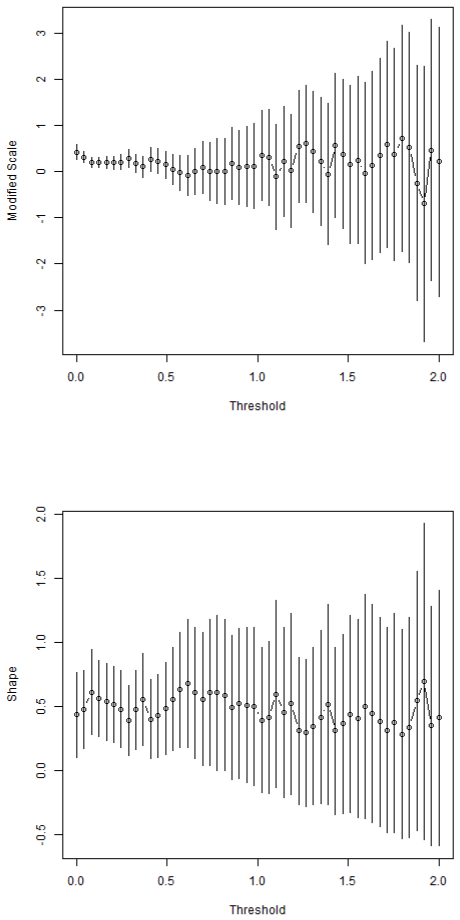
182 exceedances



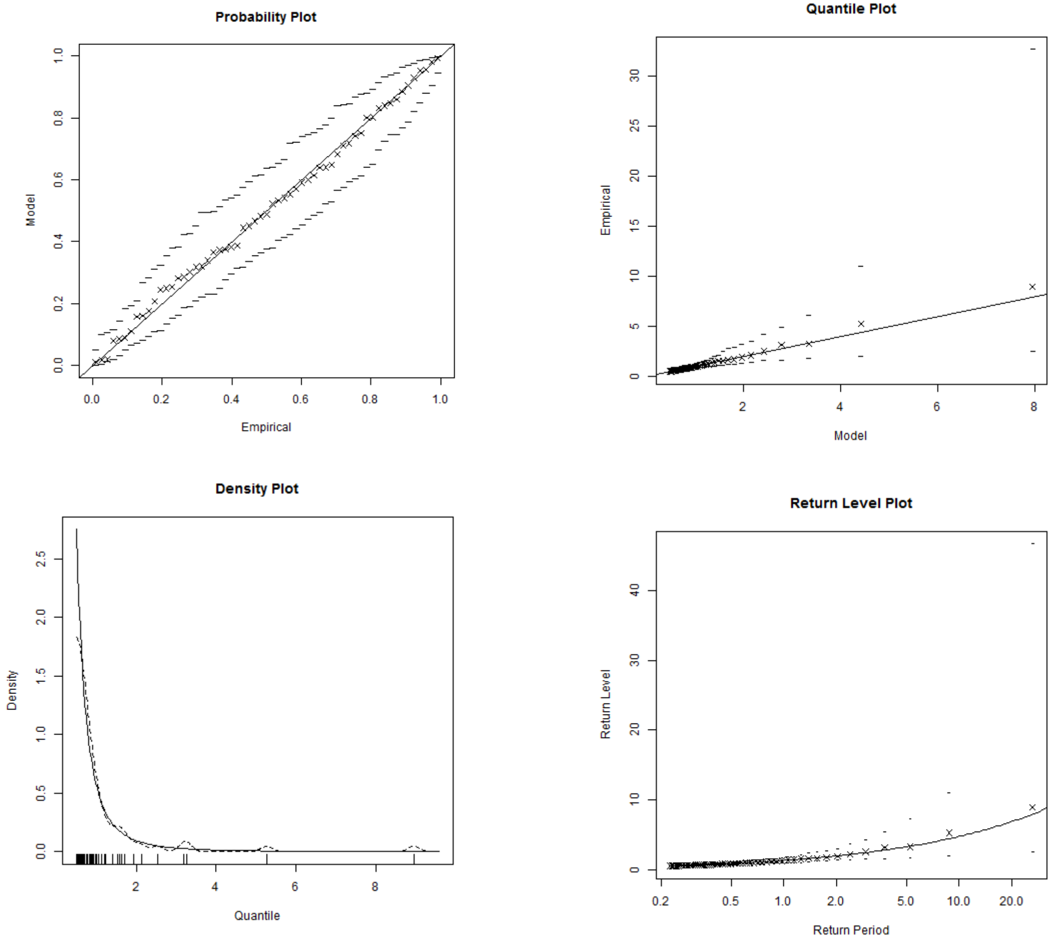
59 exceedances

Empirical Results

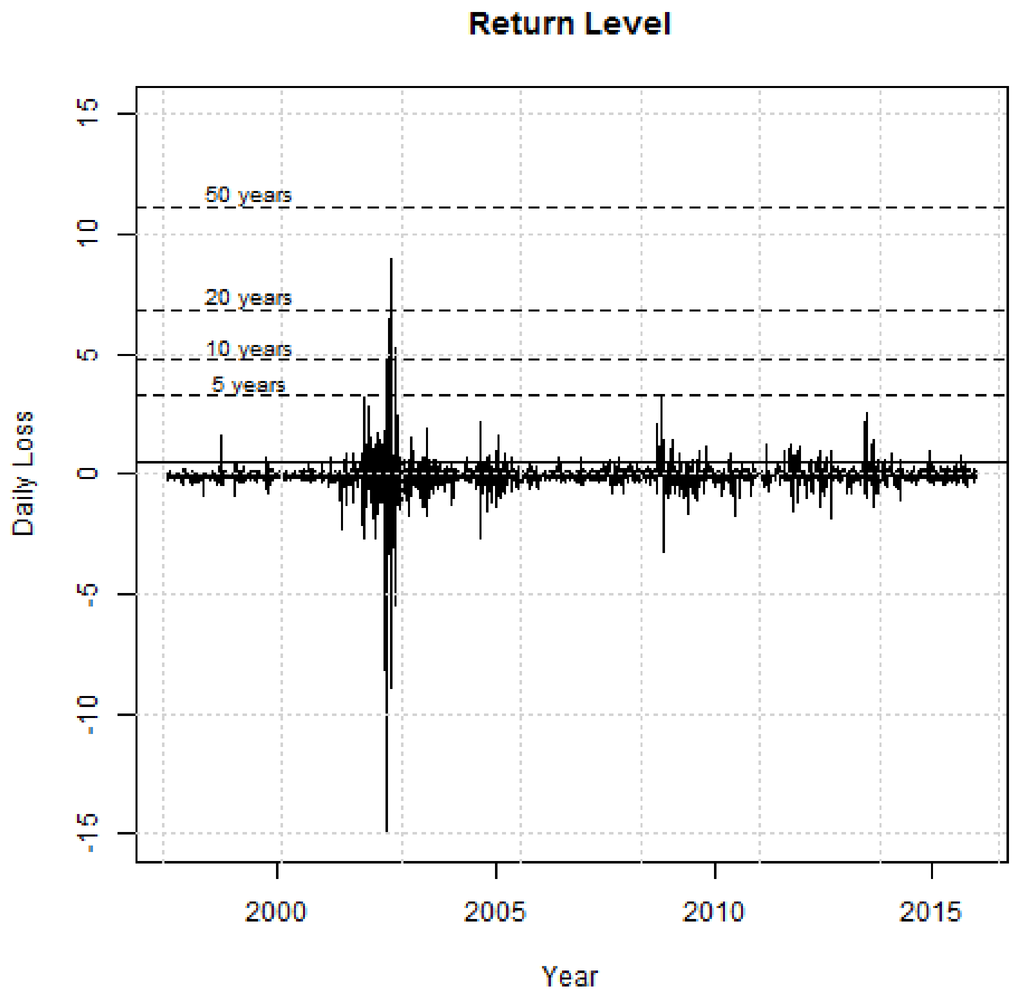
Selection of the threshold → $u = 0,5$



Diagnostic plots



Empirical Results



Return Level

5 years	**3,26%**
10 years	**4,75%**
20 years	**6,88%**
50 years	**11,14%**

Empirical Results

The performance of the different methods can be evaluated by comparing the estimates with the actual losses observed.

high variance | heavy tail
overestimates | underestimates

Value at Risk: one day horizon				
	$\alpha = 5\%$	$\alpha = 2.5\%$	$\alpha = 1\%$	$\alpha = 0.5\%$
Normal model	0.808	0.975	1.169	1.301
GPD model	0.408	0.666	1.185	1.777
Empirical Result	0.397	0.664	1.201	1.769
Expected Shortfall: one day horizon				
	$\alpha = 5\%$	$\alpha = 2.5\%$	$\alpha = 1\%$	$\alpha = 0.5\%$
Normal model	1.030	1.175	1.348	1.468
GPD model	1.049	1.583	2.658	3.887
Empirical Result	0.991	1.468	2.314	3.134

underestimates
absence of extreme values

Conclusion

- Our major conclusión is that the POT model can be useful for assessing the **size** of extreme events.
- VaR approaches based on the normal assumption of normal distribution are definitely **overestimating** low percentiles, and **underestimating** high percentiles.
- The absence of extreme values in the assumption of normal distribution **underestimate** the ES estimation for high percentiles.
- The POT model seems **coherent** with respect to the actual losses observed and is **easy** to implement.

References

Books

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Thank you!!!