An Application of Extreme Value Theory for Measuring Financial Risk in the Uruguayan Pension Fund

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Introduction

• Traditional statistical methods for financial risk measures fits models to all data even if primary focus is on extremes.

• In literature → normal assumption for financial returns.

• This assumption provides a good approximation for the average of financial returns but does not provide a good fit for the tail of the distribution.

• The last 20 years → instabilities in financial markets → volatility.

• Search for more appropriate methodologies→ Extreme Value Theory.
Risk Measures

Suppose a random variable $X$ with continuous distribution function $F$ models losses on a certain financial portfolio over a certain time horizon.

- **Value at Risk:**
  \[ \text{VaR}_\alpha = F^{-1}(1 - \alpha) \]

- **Expected Shortfall:**
  \[ \text{ES}_\alpha = E(X \mid X > \text{VaR}_\alpha) \]

- **Return Level:**
  \[ R_k^m = H^{-1}(1 - 1/m) \]

If $H$ is the distribution of the maximum observed over a successive non overlapping periods of equal length. For example, assuming a model for the annual maximum, the 15-years return level $R_{365}^{15}$ is on average only exceeded in one year out of every 15 years.
Extreme Value Theory

The Peak Over Threshold (POT) method, considers the distribution of exceedances over a certain threshold.

The distribution function $F_u$ is called the excess distribution function and is:

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F(u + y) - F(u)}{1 - F(u)}$$
Extreme Value Theory

**Theorem Pickands (1975), Balkema and de Haan (1974).**

For a large class of underlying distribution $F$, the excess distribution function $F_u$ can be approximated by GPD for increasing threshold $u$.

$$F_u(y) \approx G_{\xi, \beta}(y), \quad u \to \infty$$

where $G_{\xi, \beta}$ is the Generalized Pareto Distribution (GPD) which is given by

$$G_{\xi, \beta}(y) = \begin{cases} 
(1 + \frac{\xi y}{\beta})^{-1/\xi} & \xi \neq 0 \\
1 - e^{-y/\beta} & \xi = 0
\end{cases}$$

for $y \in [0, (x_F - u)]$ if $\xi \geq 0$ and $y \in [0, -\beta/\xi]$ if $\xi < 0$. 

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Risk Measures under Extreme Value Theory

Assuming a GPD function for the tail distribution, if we denote \( x = u + y \) then

\[
F(x) = (1 - F(u))F_u(y) + F(u)
\]

and replacing \( F_u \) by the GPD and \( F(u) \) by the empiric estimate \((n - N_u)/n\), where \( n \) is the total number of observations and \( N_u \) the number of observations above the threshold \( u \).

\[
VaR_\alpha = u + \frac{\beta}{\xi} \left( \left( \frac{n}{N_u} \right)^{-\xi} - 1 \right)
\]

\[
ES_\alpha = \frac{VaR_\alpha}{1 - \xi} + \beta - \xi u \frac{1}{1 - \xi}
\]
Empirical Results

We work with the daily losses series of AFAP SURA NAV over a period of 18 years (1997-2015). Containing 4,802 trading days. Don’t appear to have a normal distribution.
Empirical Results

In practice, we have to consider two important aspects, the selection of the threshold $u$ and the Independence of the exceedances.

$u = 0.5$

182 exceedances

59 exceedances
Empirical Results

Selection of the threshold \( u = 0.5 \)

Diagnostic plots
Empirical Results

Return Level

<table>
<thead>
<tr>
<th>Year</th>
<th>Return Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>3.26%</td>
</tr>
<tr>
<td>10 years</td>
<td>4.75%</td>
</tr>
<tr>
<td>20 years</td>
<td>6.88%</td>
</tr>
<tr>
<td>50 years</td>
<td>11.14%</td>
</tr>
</tbody>
</table>
Empirical Results

The performance of the different methods can be evaluated by comparing the estimates with the actual losses observed.

<table>
<thead>
<tr>
<th>Value at Risk: one day horizon</th>
<th>$\alpha = 5%$</th>
<th>$\alpha = 2.5%$</th>
<th>$\alpha = 1%$</th>
<th>$\alpha = 0.5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal model</td>
<td>0.808</td>
<td>0.975</td>
<td>1.169</td>
<td>1.301</td>
</tr>
<tr>
<td>GPD model</td>
<td>0.408</td>
<td>0.666</td>
<td>1.185</td>
<td>1.777</td>
</tr>
<tr>
<td>Empirical Result</td>
<td>0.397</td>
<td>0.664</td>
<td>1.201</td>
<td>1.769</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Shortfall: one day horizon</th>
<th>$\alpha = 5%$</th>
<th>$\alpha = 2.5%$</th>
<th>$\alpha = 1%$</th>
<th>$\alpha = 0.5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal model</td>
<td>1.030</td>
<td>1.175</td>
<td>1.348</td>
<td>1.468</td>
</tr>
<tr>
<td>GPD model</td>
<td>1.049</td>
<td>1.583</td>
<td>2.658</td>
<td>3.887</td>
</tr>
<tr>
<td>Empirical Result</td>
<td>0.991</td>
<td>1.468</td>
<td>2.314</td>
<td>3.134</td>
</tr>
</tbody>
</table>

- **Overestimates** (high variance)
- **Underestimates** (heavy tail)
- **Absence of extreme values**
Conclusion

• Our major conclusión is that the POT model can be useful for assessing the size of extreme events.

• VaR approaches based on the normal assumption of normal distribution are definitely overestimating low percentiles, and underestimating high percentiles.

• The absence of extreme values in the assumption of normal distribution underestimate the ES estimation for high percentiles.

• The POT model seems coherent with respect to the actual losses observed and is easy to implement.
References

Books


Articles


Thank you!!!