On four documents of Julio Garavito on actuarial sciences

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19^{th} Insurances maritime and fluvial companies Colombia

1858: El Porvenir de las Familias (Chilean)
1878: Cia. Colombiana de Seguros (Colombian)
around 1850 Foreigners mentioned (New York)

Insurances maritime and fluvial companies Colombia

An antique 64 pages document entitled ´´General conditions that the Compañía Colombiana de Seguros establish for maritime and fluvial insurances"

Documents XIX century

Compañia Colombiana de Seguro Constitution articles ´´Valuation of the policy "pp. 7-8 and Chapter 8 ´´On the insurable value"pp. 8-9.

Documents XIX century

´´the imaginary usefulness"(sic)
´´provisionaly valued policy .º ´´floating policy refering to objects that are exposed to voyage risks ´´valuation on freight"

Documents XIX century

Reimbursment of prime or premium minus bonus ´´abono por ristorno"

Documents XIX century

´´ Arbitrament Trial between insurers who embarked exporting fruits which were lost during the shipwreck of the steamship Francia-Elena and the Compañía Colombiana de Seguros" (1878)

Life Insurances 1901

Sociedad Nacional de Seguros
branch of Compañia Colombiana de Seguros

Four Documents by Julio Garavito

1. Fundamentals of insurance companies (1898) according to *Catalogo documental del Observatorio Astronomico Nacional*; or (around 1901) according dates refering to the foundation of Sociedad Colombiana de Seguros.

Four Documents by Julio Garavito

1. Fundamentals of insurance companies (1898) according to *Catalogo documental del Observatorio Astronomico Nacional*; or (around 1901) according dates refering to the foundation of Sociedad Colombiana de Seguros.
2. Calculation of premiums and reserves of life insurances (1902-1903). Archives FASECOLDA, Bogotá

Four Documents by Julio Garavito

3 Real estate developer coop. Problem(1911). Notebooks, Observatorio Astronomico Nacional

Four Documents by Julio Garavito

3 Real estate developer coop. Problem(1911). Notebooks, Observatorio Astronomico Nacional

4 Agricultural insurance (1919). Published 1939, Revista Anales de Ingenieria

Fundamentals of insurance companies 1898-1902

Preliminary notions

- Theory of annuities (sec.2).

Fundamentals of insurance companies 1898-1902

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Fundamentals of insurance companies 1898-1902

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Fundamentals of insurance companies 1898-1902

Preliminary notions

- Theory of annuities (sec.2).
- Calculation of single premium (sec.3).
- Transformation of single into yearly premiums (sec. 4).
- Calculation of combinations of premiums (sec. 5) .

Fundamentals of insurance companies

Life insurances embraces all of the operations based on observations made upon human life
The insurance company commits to paying a certain amounts under given conditions:

1. the death of the insured.
2. at determined moments given that the insured is alive or in case the insured died
3. upon completing a combination of the above cases and being the insurance taken on one or two heads.

Fundamentals of insurance companies

1. Curve of mortality, $y = f(x)$.
2. The velocity of mortality represented by angular coefficient $\frac{dy}{dx}$; rate of mortality

$$\tau = -\frac{dy}{y\,dx} \quad \text{also} \quad \tau = \frac{-f'(x)}{f(x)}.$$

Fundamentals of insurance companies

$$\frac{dy}{y} = -aq^x\, dx,$$

$$\mathrm{NepLog}(y) = -\frac{a}{l'q}q^x + c$$

or else

$$y = e^{-\frac{a}{l'q}q^x + c} = e^c e^{-\frac{a}{l'q}q^x},$$

Fundamentals of insurance companies

being $l'q = lognepq$
Naiper Logarithm, defined:

$$lognep(x) = \frac{\log \frac{10^7}{x}}{\log \frac{10^7}{10^7-1}}$$

.

$$y = A(B)^{q^x}.$$

$$y = \frac{k}{p^x}(Q)^{q^x}$$

Fundamentals of insurance companies

The probability that an individual age a die during $[x, x + dx]$ will be the product of the probabilities $x + dx$, será el producto de las probabilidades

$$\overline{w} = \frac{f(x)}{f(a)}$$

of living to age x and not reaching age $[x + dx]$

$$\overline{w}\,' = 1 - \frac{f(x + dx)}{f(x)}$$

Hence

$$\overline{w}\,\overline{w}\,' = \frac{f(x)}{f(a)}\left(1 - \frac{f(x + dx)}{f(a)}\right) = -\frac{d\,f(x)}{f(a)}. \tag{1}$$

Fundamentals of insurance companies

Annuities

Let Q_a^n the value of one *peso* to be paid by a person, present age a within n years in case he is alive.

Let us suppose w to be the limit of human life X_a the life annuity is

$$X_a = Q_a^1 + Q_a^2 + \cdots + Q_a^{w-a} = \sum_i^{w-a} Q_a^i.$$

Fundamentals of insurance companies

Teorema

$$X_a = Q_a^1\left(1 + X_{a+1}\right) \tag{43}$$

Fundamentals of insurance companies

Deferred annuity n years

$X_a{}^n = Q_a^n X_{a+n}..$

Fundamentals of insurance companies

${}_nX_a$ be the **Temporary annuity** the difference between the annuity X_a and the deferred annuity ${X_a}^n$:

$$ {}_nX_a = X_a - Q_a^n X_{a+n}. \tag{2} $$

Fundamentals of insurance companies

Single premiums Let P_a be the present value of 1 *peso*, the current worth of future 1 *peso* that will be paid at the occurrence of the insured's death, age a,

$$P_a = -\int_{x=0}^{x=\infty} (1+t)^{-x} \frac{d\, f(x)}{f(a)}.$$

Fundamentals of insurance companies

$$P_a = -\left[(1+t)^{-x}\frac{f(a+x)}{f(a)}\right]_0^{\infty} - \int_0^{\infty}\frac{f(a+x)}{f(a)}(1+t)^{-x}\,\mathrm{NepLog}(1+t)\,dx$$

o bien

$$P_a = 1 - \mathrm{NepLog}(1+t)\int_0^{\infty}\frac{f(a+x)}{f(a)}(1+t)^{-x}\,dx.$$

Fundamentals of insurance companies

Llamemos I la integral; tendremos

$$\sum_{0}^{\infty} \frac{f(a+x)}{f(a)}(1+t)^{-x} > I > \sum_{1}^{\infty} \frac{f(a+x)}{f(a)}(1+t)^{-x}$$

Fundamentals of insurance companies

The integral I is called continous annuity
its approximate value is

$$I = X_a + \frac{1}{2} - \frac{1}{12}(\tau + \rho)$$

with τ, the continous rate of mortality and ρ the continous interest rate t.

Fundamentals of insurance companies

$$\sum_{0}^{\infty} \frac{f(a+x)}{f(a)}(i+t)^{-x} = \sum_{0}^{\infty} \frac{f(a+i)}{f(a)}(i+t)^{-i} = \sum_{0}^{\infty} Q_a^i = 1 + X_a$$

y

$$\sum_{1}^{\infty} \frac{f(a+x)}{f(a)}(i+t)^{-x} = \sum_{1}^{\infty} \frac{f(a+i)}{f(a)}(i+t)^{-i} = \sum_{1}^{\infty} Q_a^i = X_a.$$

Así pues

$$1 + X_a > I > X_a.$$

Fundamentals of insurance companies

Garavito pointed out that the integral

$$\int_0^{\infty} \frac{f(a+x)}{f(a)}(1+t)^{-x}$$

has not been able to be solved. (The theorems on impossibility dated back to *J. Liouville* 1834 - 1850, $f(x) = Ke^{Be^{Mx}}$ to forms $\int \frac{e^{-Z}}{Z} dZ$, $\int \frac{e^{Z}}{Z} dZ$, *Abramowitz an Stegun (1970), Handbook of mathematical functions* p.228, 245-249)

Fundamentals of insurance companies

$$P_a = 1 - \mathrm{NepLog}(1+t)\left[X_a + \frac{1}{2} - \frac{1}{12}(\tau+\rho)\right]. \qquad (3)$$

Fundamentals of insurance companies

$$t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots = t\left(1 - \frac{t}{2} + \frac{t^2}{3} - \cdots\right)$$

will bring

$$P_a = 1 - \left(1 - \frac{t}{2} + \frac{t^2}{3} - \cdots\right) t\left(X_a + \frac{1}{2} - \frac{1}{12}(z+\rho)\right).$$

Fundamentals of insurance companies

By approximation methods the formulae are easy to be obtained according the annuity
is to be paid at the begining, at the middle or at the end of the year:

$$P_a = 1 - tX_a \qquad \text{(at the begin)} \qquad (4)$$

$$P_a = \frac{1 - tX_a}{(1+t)^{\frac{1}{2}}} \qquad \text{(at the middle)} \qquad (5)$$

$$P_a = \frac{1 - tX_a}{1+t} \qquad \text{(at the end)} \qquad (6)$$

Fundamentals of insurance companies

- Life Insurances for the whole life:
  i) on one head, ii) on more than one head.
- Temporary : i) ordinary, ii) yearly insurance.
- mixed insurances: i) normal, ii) fixed-term .
- Life insurancesa: i) for a fixed amount capital, ii) for an annuity income.

Fundamentals of insurance companies

First case: Payment of a unique premium.
If the insured pay a unique premium, the amount of the premium must equal the amount of capital that would be paid at the occurrence of the death:

$$P_a = 1 - \left[X_a + \frac{1}{2} - \frac{1}{12}(z+\rho)\right] \mathrm{NepLog}(1+t),$$

the formula (3), if we suppose the payment is to be done at the moment of the death ;

$$P_a = 1 - tX_a,$$

the formula (4), if it is supposed that it is payed or it is at the company's stock to the disposal of the insured from the beginning of the year in which the death occurs

$$P_a = \frac{1 - tX_a}{1+t},$$

the formula (6), if it is supposed the payment is to be done at the end of

Fundamentals of insurance companies

Second case: Payment by yearly constant premiums during whole life (ordinary life policy in Sociedad Colombiana de Seguros).
The yearly premium is calculated by transformation of the single premium into yearly premiums along the life of the insured Let a the age of the insured, p_a the yearly premium to be found. The first premium being payable beforehand, the present value of the set of premiums, the present commitment of the insured being $p_a(1+X_a)$; this one being equivalent to the single premium P_a, then

$$p_a = \frac{P_a}{1+X_a}. \tag{7}$$

If formula (6) holds we would have

$$p_a = \frac{1-tX_a}{(1+t)(1+X_a)}. \tag{8}$$

Fundamentals of insurance companies

Third case: Premiums to be paid during a limited term (policy of the Compañía Colombiana de Seguros).
Instead of paying a single premium, the insured is able to pay a temporary premium i.e., it can be required to be paid on the part of the insured for a number of years only and stopping the payments or upon the ocurrence of death of the insured.
Let n be the fixed number of years and ${}_np_a$ the temporary prermium to be found: The present value of the set of premiums is ${}_np_a(1+{}_{n-1}X_a)$ being ${}_{n-1}X_a$ the temporary annuity (2) corresponding to $n-1$ years. This amount must equal the single premium P_a, hence

$$ {}_np_a = \frac{P_a}{1+{}_{n-1}X_a}. \tag{9} $$

By substituing (6) P_a and ${}_{n-1}X_a$, we get

$$ {}_np_a = \frac{1-tX_a}{(1+t)(1+X_a-Q_a^{n-1}X_{a+n-1})}. \tag{10} $$

Fundamentals of insurance companies

Premium rate the premium rate is generally calculated by adding to the constant yearly premium (8) or temporary (10) a charge 10 %. The average income of the operation will be obtained by multiplying the actual charge (the difference between yearly and pure premium) times the continous or temporary annuity to be payed beforehand, i.e. by $(1 + X_a)$ o por $(1 +_{n-1} X_a)$ según el caso.

The insurance on two heads will not be studied

Fundamentals of insurance companies

Temporary insurance: the company has the commitment to pay a capital A at the moment of ocurrence of the insured's death whenever this happen in a period of time. Upon the expiration of the agreed time and the surviving of the insured the invested premiums will be owned by the company

Single premium.

the single premium, P_a, must equal the amount of the commitmet, i. e., the difference between the price of a life insurance for the whole life at age $a + n$ being this last amount lesser than its actual value, i. e., multiplyed by Q_a^n the present value of one *peso* that will be paid by a person age a within n years.

Fundamentals of insurance companies

therefore,

$$ {}_nP_a = P_a - P_{a+n}Q_a^n. \tag{11} $$

Fundamentals of insurance companies

Yearly premium.
It is obtained by transformation of the single premium ${}_nP_a$ into temporary yearly premiums:

$$ {}_np_a = \frac{{}_nP_a}{1 +_{n-1} X_a}. \tag{12}$$

We will not examine this insurance for it has not been used in Sociedad Colombiana de Seguros.

Fundamentals of insurance companies

Mixed insurance it is a contract the company will pay an amount in case the insured death ensue in a certain pre-fixed period or at the end of the period if the insured is alive It can be given in two manners
i) type 1 when the capital to be paid is the same in case of life than in case of death
ii) type 2 when the capital to be paid is different in case of life than in case of death

Fundamentals of insurance companies

i) type 1. If the amount of \$1 is paid in any case. calling ${}_nP_a{}'$ the single premium then

$$ {}_nP_a{}' = P_a - P_{a+n}Q_a^n + Q_a^n. \tag{13} $$

Fundamentals of insurance companies

The yearly premium ${}_np_a{}'$ is obtained by the transformation of the single one ${}_nP_a{}'$ into temporary yearly premiums

$$ {}_np_a{}' = \frac{{}_nP_a{}'}{1 + {}_{n-1}X_a}. $$

Fundamentals of insurance companies

ii) type 2. Let A the payable capital if death occured and B the payable capital in case of surviving

Fundamentals of insurance companies

The value of \$1 out of the capital A is worth

$$ {}_nP_a'' = P_a - P_{a+n}Q_a^n + \frac{B}{A}\,Q_a^n $$

or

$$ {}_nP_a'' = P_a + \left(\frac{B}{A} - P_{a+n}\right) Q_a^n. \tag{14} $$

Fundamentals of insurance companies

yearly temporary premiums

$$ {}_{n}p_{a}'' = \frac{{}_{n}P_{a}''}{1 + {}_{n-1}X_{a}}. \tag{15} $$

Fundamentals of insurance companies

The problem has a lot of possible variations that we will not consider

Fundamentals of insurance companies

Calculation of reserves

Fundamentals of insurance companies

After considering a group of policies during the first years, there will be a considerable surplus from received premiums. The mass of premiums decrease while the sinisters increase, therefore there will be a point where, for a fixed number of policies the number of premiums is less than the amount of sinisters.
Unlike maritime companies a year's surplus cannot be considered as a benefit.

Fundamentals of insurance companies

The amount of the difference is to be to filled up by the reserve.
We will call it reserve for risks in progress and it is defined as:

> *The reserve on a policy in progress is an amount equal to the surplus of the present value of the commitment of the company taken on the present value of the insured's commitment*

Fundamentals of insurance companies

Reserve Ordinary Policy Reserve m years after the contract ${}_mV_a$ (for 1 peso):
the amount of the company's commitment

$$P_{a+m} = \frac{1 - tX_{a+m}}{1 + t}.$$

Fundamentals of insurance companies

The commitment for the insurer is worth the yearly constant premium p_a taken on a person age $a+m$ hence the reserve is:

$$ {}_mV_a = P_{a+m} - p_a(1 + X_{a+m}) \tag{16} $$

or

$$ {}_mV_a = 1 - \frac{1 + X_{a+m}}{1 + X_a}. \tag{17} $$

Fundamentals of insurance companies

Reserve limited payment policy The reserve ${}_mV_{(a,n)}$ is calculated: The insurance has been contacted at age a thorough a payment of premiums during n years and let us suppose m years have gone by n. The commitment of the company amounts to

$$P_{a+m} = \frac{1 - tX_{a+m}}{1+t};$$

ant the commitmet of the insured consists of the payment of $n-m$ premiums ${}_np_a$, so it is equivalent to

$${}_np_a(1 +_{n-m-1} X_{a+m}) = \frac{1+tX_a}{1+t} \cdot \frac{1 +_{n-m+1} X_{a+m}}{1 +_{n-1} X_a}.$$

The reserve is then the difference

$${}_mV_{(a,n)} = P_{a+m} -_n p_a(1 +_{n-m-1} X_{a+m}). \tag{18}$$

Fundamentals of insurance companies

Reserve Dotal policy

By calling a the age of the insured and n the term 10, 15 o 20 years ; (14) y (15) reduce to (by shifting the notationt ${}_nP_a{}''$ by $P''_{(a,n)}$ and ${}_np_a{}''$ by $p''_{(a,n)}$

Single premium $P''_{(a,n)} =_n P_a{}''$ (14)

$$P''_{(a,n)} = \frac{1}{1+t}\left\{\left[\frac{B}{A}(1+t) - 1 + tX_{a+n}\right] Q_a^n + 1 - tX_a\right\}. \qquad (19)$$

Temporary yearly premium $p''_{(a,n)} =_n p_a{}''$ (15)

$$p''_{(a,n)} = \frac{P''_{(a,n)}}{1 + X_a - Q_a^{n-1} X_{a+n-1}}. \qquad (20)$$

The reserve of the aforementioned policy is calculated taking into account: the mixed insurance has been contracted at age a during n years, then the company's commitment let us say ${}_mC_{(a,n)}$ after m years will be the single premium of the mixed insurance made at age $a+m$ for $n-m$ years, i. e.

Fundamentals of insurance companies

Apendix Annuities on 2 heads
Continous annuity on 2 heads
Deferred annuity n years on 2 heads
Temporary annuity n years on 2 heads
Annuity on 2 heads payable upon acurrence of the last death
Yearly premiums payable upon ocurrence the first death

Fundamentals of insurance companies

$$Q^n_{a,b} = (1+t)^{-n} \frac{f(a+n)}{f(a)} \frac{f(b+n)}{f(b)} \tag{24}$$

Fundamentals of insurance companies

and

$$X_{a,b} = \sum Q^j_{a,b},$$

$$Q^n_{a+1,b+1} = \frac{Q^{n+1}_{a,b}}{Q^1_{a,b}};$$

then

$$X_{a,b} = Q^1_{a,b}\left(1 + X_{(a+1)(b+1)}\right). \tag{25}$$

Fundamentals of insurance companies

Continous annuity on two heads:

$$\overline{X}_{a,b} = \int_0^\infty \frac{f(a+x)\, f(b+x)}{f(a)\, f(b)} (1+t)^{-x}\, dx = X_{a,b} + \frac{1}{2} - \frac{1}{12}(z_a + \tau_a + \rho). \tag{26}$$

Engineer J. Garavito

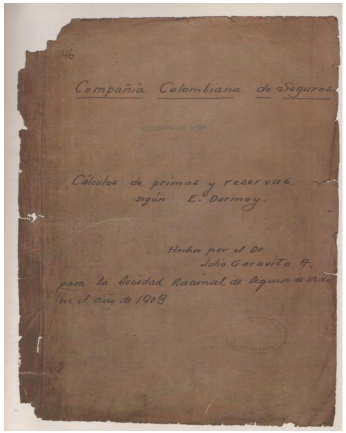
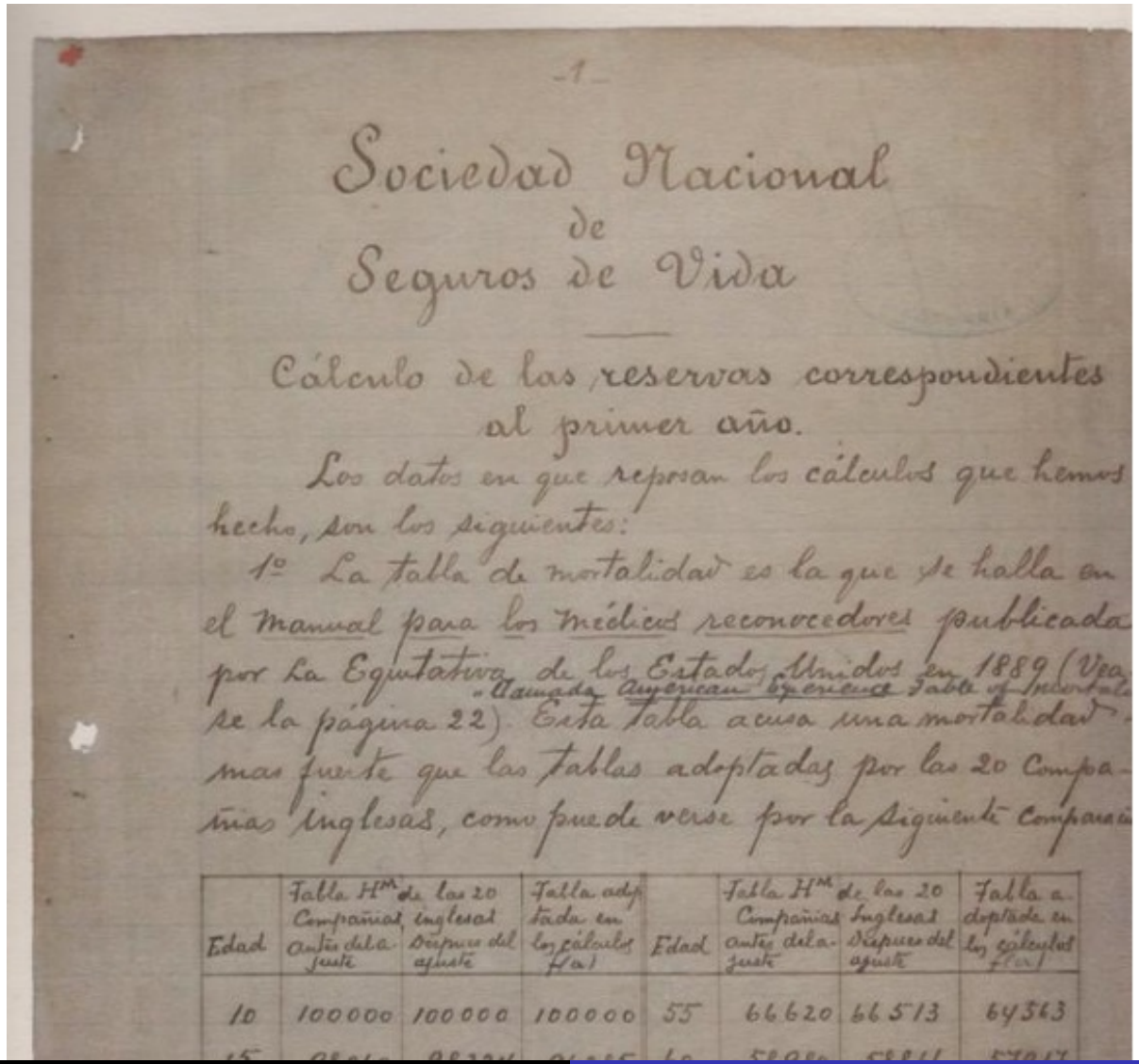


Figura: Archives FASECOLDA (1902-1903)

Engineer J. Garavito



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Sociedad Nacional
de
Seguros de Vida

Cálculo de las reservas correspondientes al primer año.

Los datos en que reposan los cálculos que hemos hecho, son los siguientes:

1º La tabla de mortalidad es la que se halla en el Manual para los médicos reconocedores publicada por la Equitativa de los Estados Unidos en 1889 (Vea se la página 22). Esta tabla acusa una mortalidad mas fuerte que las tablas adoptadas por las 20 Compañías inglesas, como puede verse por la siguiente Comparación

Edad	Tabla H^M de las 20 Compañías inglesas antes del ajuste	Tabla H^M de las 20 inglesas después del ajuste	Tabla adoptada en los cálculos (Ha)	Edad	Tabla H^M de las 20 inglesas antes del ajuste	Tabla H^M de las 20 inglesas después del ajuste	Tabla adoptada en los cálculos (Ha)
10	100000	100000	100000	55	66620	66513	64363

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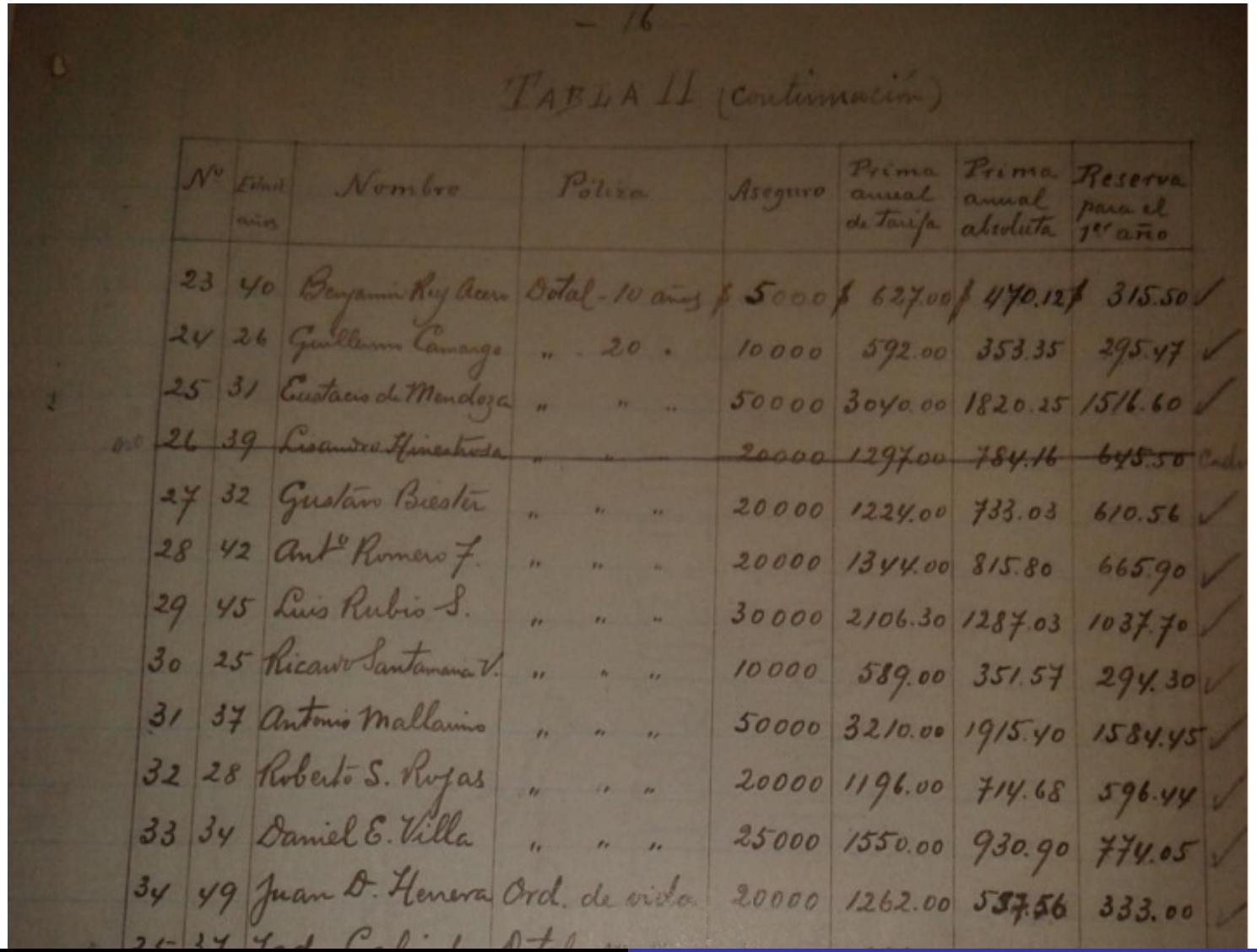

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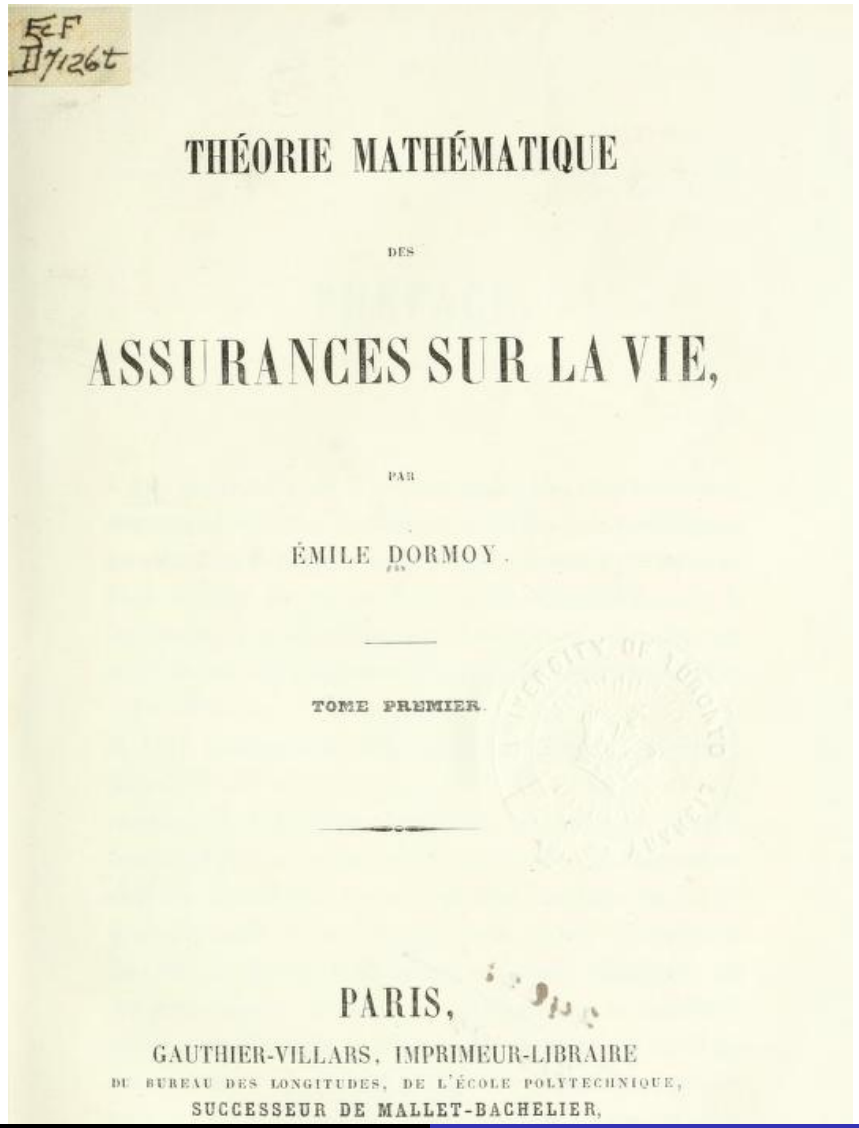
TABLA II (Continuación)

Nº	Edad años	Nombre	Póliza	Aseguro	Prima anual de tarifa	Prima anual absoluta	Reserva para el 1er año	
23	40	Benjamín Rey Acero	Dotal - 10 años	$ 5000	$ 627.00	$ 470.12	315.50	✓
24	26	Guillermo Camargo	" 20 "	10000	592.00	353.35	295.47	✓
25	31	Eustacio de Mendoza	" " "	50000	3040.00	1820.25	1516.60	✓
26	39	~~Lisandro Hinestrosa~~	~~" " "~~	~~20000~~	~~1297.00~~	~~784.16~~	~~645.50~~	Cada
27	32	Gustavo Biester	" " "	20000	1224.00	733.03	610.56	✓
28	42	Antº Romero F.	" " "	20000	1344.00	815.80	665.90	✓
29	45	Luis Rubio S.	" " "	30000	2106.30	1287.03	1037.70	✓
30	25	Ricardo Santamaría V.	" " "	10000	589.00	351.57	294.30	✓
31	37	Antonio Mallarino	" " "	50000	3210.00	1915.40	1584.45	✓
32	28	Roberto S. Rojas	" " "	20000	1196.00	714.68	596.44	✓
33	34	Daniel E. Villa	" " "	25000	1550.00	930.90	774.05	✓
34	49	Juan D. Herrera	Ord. de vida	20000	1262.00	537.56	333.00	✓

Main Reference Text, Emile Dormoy (archive.org)



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THÉORIE MATHÉMATIQUE

DES

ASSURANCES SUR LA VIE,

PAR

ÉMILE DORMOY.

TOME PREMIER.

PARIS,

GAUTHIER-VILLARS, IMPRIMEUR-LIBRAIRE

DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE,

SUCCESSEUR DE MALLET-BACHELIER,

Calculation of premiums and reserves of life insurances (1902-1903). Fasecolda Archives

Reference text *Thèorie mathemàtique des assuranses sur la vie, Dormoy (1878). París. Gauthier Villars*.

Calculation of premiums and reserves of life insurances

Given 167 policies of individuals he proposes to calculate the corresponding premiums and reserves

Calculation of premiums and reserves of life insurances

a) There are calculation for policies numbered 1 to 45 and to 69 to 105 Sociedad Nacional de Seguros , Garavito (1902). Also includes two pages with scratch calculations and 4 lines with policies scratched out

Calculation of premiums and reserves of life insurances

b) Garavito explains numerical calculation with logarithms were made together with Pedro de Francisco.

c) The information in the policies table includes insurer's age, name, type of policy (dotal to 10, 15, 20 years; ordinary and there is one policy to limited payments) the amount of the insurance, the yearly premium, the net premium, the reserve for the first year.

Calculation of premiums and reserves of life insurances

d) According to page numbering it seems one page with policies 47 to 68 is lost.
There is a note that explains that 5 policies were not calculated because they does not fulfill some basic parameters

Calculation of premiums and reserves of life insurances

e) second table (*Garavito (1902)* p.20-25 there are policies numbered, 1 to 167, calculation of reserves to the end of 1903. The table includes type of policy (two dotal - five years, 2 ten years limited policies and the others are 10-15 or 20 dotal and ordinary payment policies

Calculation of premiums and reserves of life insurances

f) There is a reference *Prospecto de la Sociedad [Nacional de Seguros]* non included in the notes.

g) All of the insurers are men ¡¡

Calculation of premiums and reserves of life insurances

- Notation as in Dormoy and also as in Fundamentals of insurance companies

Calculation of premiums and reserves of life insurances (1902-1903). Fasecolda Archives

American Equivalence (reference also Equitativa) 1898
Table of Mortality of the 20 english companies
Dormoy's table H^M
Handbook of examiner physicians

Calculation of premiums and reserves of life insurances

Garavito's reference is possibly to The Equitable and American Experience 1898
American Experience included the physicians' guide

Calculation of premiums and reserves of life insurances

Garavito's main reference is Dormoy's book in which there are also references to Depacieux and Duvillard tables

Edad	Tabla H^M sin ajuste.	Tabla H^M ajustada	Tabla adoptada
10	100000	100000	100000
15	98060	98224	96285
20	96160	96223	92637
25	92970	93061	89032
30	89870	89865	85441
35	86250	86281	81822
40	82230	82284	78106
45	77930	77919	74173
50	72740	72726	69804
55	66620	66513	64363
60	58980	58866	57917
65	49310	49297	49341
70	37930	38124	38569
75	25470	25691	26237
80	13920	13930	14474
85	5410	5422	5485
90	1500	1460	847
95	135	3	

J. Garavito

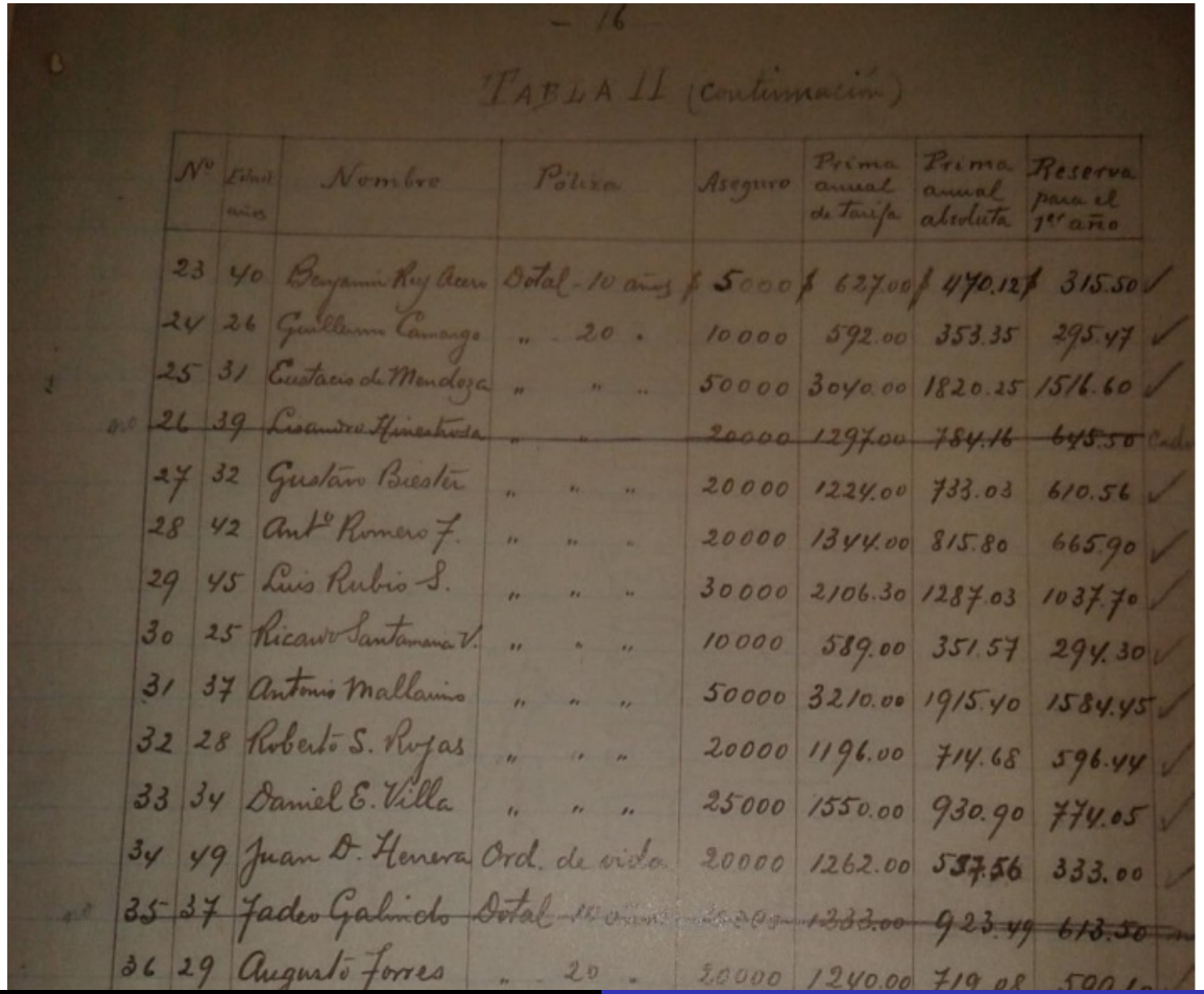

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TABLA II (Continuación)

Nº	Edad años	Nombre	Póliza	Asegurо	Prima anual de tarifa	Prima anual absoluta	Reserva para el 1er año	
23	40	Benjamín Rey Acero	Dotal-10 años	$ 5000	$ 627.00	$ 470.12	$ 315.50	✓
24	26	Guillermo Camargo	" 20 "	10000	592.00	353.35	295.47	✓
25	31	Eustacio de Mendoza	" " "	50000	3040.00	1820.25	1516.60	✓
~~26~~	~~39~~	~~Lisandro Hinestrosa~~	~~" " "~~	~~20000~~	~~1297.00~~	~~784.16~~	~~645.50~~	
27	32	Gustavo Briesler	" " "	20000	1224.00	733.03	610.56	✓
28	42	Antº Romero F.	" " "	20000	1344.00	815.80	665.90	✓
29	45	Luis Rubio S.	" " "	30000	2106.30	1287.03	1037.70	✓
30	25	Ricardo Santamaría V.	" " "	10000	589.00	351.57	294.30	✓
31	37	Antonio Mallarino	" " "	50000	3210.00	1915.40	1584.45	✓
32	28	Roberto S. Rojas	" " "	20000	1196.00	714.68	596.44	✓
33	34	Daniel E. Villa	" " "	25000	1550.00	930.90	774.05	✓
34	49	Juan D. Herrera	Ord. de vida	20000	1262.00	537.56	333.00	✓
~~35~~	~~37~~	~~Jadeo Galindo~~	~~Dotal 10 años~~	~~5000~~	~~1333.00~~	~~923.49~~	~~613.50~~	
36	29	Augusto Torres	" 20 "	20000	1240.00	719.08	590.10	✓

Calculation of premiums and reserves of life insurances (1902-1903). Fasecolda Archives

Reference text *Thèorie mathemàtique des assuranses sur la vie, Dormoy (1878). París. Gauthier Villars*.
$Log(Q_a^1)$= logaritm of the value of one peso to be paid by a person who is a within one year in case he is alive
$LogQ_a^n$ the logaritm of the value of one peso to be paid by one person who is a within n years if alive.
$LogX_a$

$$1 + t = 1{,}06 \qquad \log(1 + t) = 0{,}02531$$

$$\log(1 + t)^{-1} = \bar{1}{,}97469$$

The old fashioned style for mantise of logaritms $\bar{1}$,97469 in tables of logaritms to simplify calculations: characteristic 1 mantise ,97469

Calculation of premiums and reserves of life insurances (1902-1903)

Theorem

$$X_a = Q_a^1\left(1 + X_{a+1}\right) \tag{43}$$

Calculation of premiums and reserves of life insurances (1902-1903)

Examples:

$$X_{94} = Q'_{94} = (1{,}06)^{-1}\frac{3}{21} = \frac{3}{21 \times 1{,}06} = 0{,}1348$$

Calculation of premiums and reserves of life insurances (1902-1903)

Examples:

$$X_{94} = Q'_{94} = (1{,}06)^{-1}\frac{3}{21} = \frac{3}{21 \times 1{,}06} = 0{,}1348$$

$$X_{36} = Q'_{36}(1 + X_{37})$$

Calculation of premiums and reserves of life insurances (1902-1903)

Examples:

$$Q'_{36} = (1{,}06)^{-1}\frac{f(37)}{f(36)} = \frac{80353}{81090} \times \frac{1}{1{,}06}$$

Example:

$$X_{36} = \frac{1}{1{,}06} \times \frac{80353}{81090} \times 13{,}653 = 12{,}763$$

Annuities

AGE	Table V	Calculated Table
25	13.6836	13.6630
30	13.2392	13.3170
35	12.7152	12.8680
40	12.0866	12.2890
45	11.3075	11.5540
50	10.3981	10.6397

Cuadro: Life annuities

Other annuities

- Deferred annuity n years.

$$X_a{}^n = Q_a^n X_{a+n}.$$

- Temporary annuity ${}_nX_a$

$${}_nX_a = X_a - Q_a^n X_{a+n}.$$

Premiums

ordinary policy example.
Policy N°34 Juan D. Herrera
Age $a = 49$; *value* $A = \$20{,}000$; *net premium* $=\$1262,00$. *We have single premium:* $P_a = \frac{1-0{,}6502}{1+0{,}06}$
$= \frac{0{,}3498}{1{,}06} = 0{,}3300$

Premiums

Yearly premium

$$p_a = \frac{0{,}3300}{11{,}837} = 0{,}027878 \tag{27}$$

Reserve:

$$_1V_a = 1 - \frac{11{,}6397}{11{,}8370} = 0{,}01665 \tag{163}$$

Premiums

Since the insurance value is \$20,000, we get:
Pure Premium $= 0{,}3300 \times 20{,}000 =$ \$6600,00,
Yearly premium $= 0{,}027878 \times 20{,}000 =$ \$557,56
Reserve First year
$= 0{,}01665 \times 20{,}000 =$ \$333,00

Premiums

Limited payments policy *Policy N°48 Hemel Florez, 38 , amount* \$12000, *limited payments to 20 years.*

$$P_a = \frac{1 - 0{,}7522}{1 + 0{,}06} = \frac{0{,}2478}{1{,}06} = 0{,}2338 \tag{28}$$

Premiums

$$ {}_{n}p_{a} = \frac{0{,}2338}{1 + 12{,}537 - 0{,}2578 \times 9{,}0805} = 0{,}02088 \tag{29} $$

$$ {}_{m}V_{(a,n)} = \frac{1 - 0{,}7450}{1{,}06} - 0{,}02088 \left(1 + 12{,}416 - \frac{1}{(1{,}06)^{18}} \frac{62104}{78862} \times 9{,}0805 \right) \tag{30} $$

$$ = 0{,}24056 - 0{,}22782 = 0{,}01274 $$

Premiums

for $m = 1$, value \$12000, we will get:
Single premium = \$2805,60
Yearly premium \$250,56.
Reserve first year \$152,88

Real estate developer coop(1911)

Notebook Problem of applied mathematics.
Calculate the number of members in the co-operative that should be necessary for the housing developer to accomplish its commitment 90 months so that it can be secure creditor of mortgages, and also it can pay the life insurances of mortgagers and the home fire insurances

Real estate developer coop (1911)

Conditions:
to draw lots for built homes periodically.
the mortager will pay 1.56 the value of the home
the mortgager will pay half the mortgage as long as he does not own the house
and full mortgage if gainer
the mortgager and the house must be insured

Real estate developer coop (1911)

Supossed
x =number of members
90*months* split in 30 units time (trimesters)
house type 1 amount to 500*pesos*
house type 2 amount to 1500*pesos*
house type 3 amount to 2500*pesos*

Real estate developer coop(1911)

Reducing home prices to type 1
3 houses (one of each type) amount to 9 homes per unit time (trimester)
1800*pesos* insurances costs-period

Real estate developer coop (1911)

$780 pesos = 26 pesos \times 30$

$c = 26 pesos = one \quad installment$

Real estate developer coop (1911)

Let us S_1 be the remainder after first period:
and $x=$ number of members
$S_1 = \frac{1}{2}cx - g - 500r$

Real estate developer coop (1911)

$\frac{S_1}{500} = 0{,}002 \times S_1$ homes for second lottery:
income $\frac{1}{2}c0{,}002 \times S_1$
let S_2 be the remainder after second period:
$S_2 = S_1 + 0{,}001 \times S_1 \times c - \kappa cr$
being $\kappa =$ coefficient for homes allotted in the lottery
$\kappa \cong \frac{5}{8} = \frac{1}{b-a}\int_a^b f(t)dt$

Real estate developer coop (1911)

$$
\begin{aligned}
S_3 &= (1+0{,}001c)S_2 - \kappa cr \\
S_3 &= (1+0{,}001c)^2 S_1 - (1+0{,}001c)b - (1+0{,}001c) \\
S_3 &= \underbrace{(1+0{,}001c)^2}\, S_1 - \underbrace{(1+0{,}001c)}\, b - \underbrace{(1+0{,}001c)} \\
S_3 &= a^2 S_1 - ab - b \\
b &:= \kappa cr
\end{aligned}
$$

Real estate developer coop (1911)

$$S_j = a^{j-1}\left(S_1 - \frac{b}{\alpha}\right) + \frac{b}{\alpha}$$
$$\alpha = 0{,}001c$$

Real estate developer coop (1911)

$$S_n = a^{n-1}\left(S_1 - \frac{b}{\alpha}\right) + \frac{b}{\alpha}$$

Real estate developer coop (1911)

$$\sum S_j = \frac{a^n - 1}{\alpha}\left(S_1 - \frac{b}{\alpha}\right) + \frac{b}{\alpha}$$

Real estate developer coop (1911)

Sum must equal the amount x homes minus 30 lotteries
$500x - 15000r$

Real estate developer coop (1911)

$$\sum S_j = \frac{a^n - 1}{\alpha}\left(S_1 - \frac{b}{\alpha}\right) + \frac{b}{\alpha} = 500x - 15000r$$

Real estate developer coop (1911)

By solving
$x = 2853$ amount to:
aprox. $951 type1 homes$ $317 type2 homes$ $190 type3 homes$

Agricultural Insurance

Submittal to the Congreso Nacional de Agricultores (1911). Published Anales de Ingenieria 1931
The said submittal points out that, so far, the Congreso Agricola has devoted itself to the perfection of agricultural methods and to both inside and outside marketing, but the topic that it proposes is that agricultural insurance be oriented by the same rules as those that guide maritime insurance, which requires knowledge of the annual average of agricultural losses caused by floods, frost, death of cattle, etc. and even though this statistics is not well-known among us, the inconvenience can be remedied by providing the company, that submits the proposal, with a form of mutual insurance.

Agricultural Insurance

Nonetheless, the most conspicuous cause that Garavito points out to explain the losses has been Wars; let us remember that since the 19th century, successive regional conflicts and the One Thousand Day War affected the country; therefore, Garavito proposed the creation of an insurance company to tackle the issues resulting from these periodic wars.

Agricultural Insurance

Garavito had noticed that wars lasted approximately one year and they took place every ten years; so landowners would lose one year of profits for every ten-year period of labor besides material damages. This amounted to a compulsory tax that amounted to ten percent on productivity and the most affected regions were those located at sea level or close thereto.

Agricultural Insurance

Then, Garavito proposed the creation of an insurance company that would distribute, among all the inhabitants of the country, the war tax that affected the agricultural sector; thus, farmers and cattle ranchers would add the tax to the price of their products. Garavito mentions that before the creation of maritime insurance, traders used to suffer losses due to shipwrecks; so they were forced to set up insurance companies to protect themselves against bankruptcy.

Agricultural Insurance

Furthermore, Garavito states that calculating the premium for an insurance policy in the agricultural sector is difficult; also, he states that the said premium could amount to three percent of the annual production of each ranch that can be added to the price the consumer pays. Additionally, he quotes: **Biekerdike Ïncipient Taxes Economic Journal, V. XVI" (no date)**.

Agricultural Insurance

Garavito concludes that the reserve of the potential company must be enormous to offset the damages in the aftermath of the War and the collection of those moneys would not cause any disturbance to the economic stability because such amounts would not remain in the vaults of the insurance companies; rather, those monies would be transferred to banks so that the latter could invest them.

Agricultural Insurance

Refering to the banking crisis that affected the United States around 1907, Garavito demonstrated how a certain amount of money is susceptible, due to currency, to represent enormous amounts. The deposit in one single bank was larger than the total amount of the nation´s budget; he also stated that, although banks could not pay, they were not bankrupt

Agricultural Insurance

Garavito suggest investing the amounts collected in the increment of productivity and in public works; he concludes by pointing out that this type of insurance be implemented in developed countries by means of an agricultural tax that would play the role of an insurance premium; he further states that the public in general does not trust that banks will be honest at the time of collecting such tax or that the said banks will do a better job than the government in that respect.

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Thanks a lot.