Unlocking reserve assumptions based on the retrospective loss random variable

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Outline of work

- Reserve is typically calculated based on the prospective random variable, e.g. the mean, but could use other risk measures.
- Historically, we have developed equivalence of a prospective reserve with that of a retrospective reserve. But we never defined a random variable corresponding to a retrospective reserve.
- Definition of a retrospective random variable: show its expectation is the same as that of the prospective random variable.
- This random variable is a powerful tool for providing us valuable historical information on the pattern and significance of deviation of actual experience from that assumed for reserving purposes.
  - Can be used as a guide whether it is necessary to adjust prospective reserves.
  - Can be used to help determine the procedure for adjusting.

Some motivation

- Reserves are funds set aside to meet insurer's future obligations; largest item on the balance sheet.
- Reserve basis and assumptions have historically been "locked-in" at policy issue, but that is changing:
  - FAS 97 and 120 for GAAP - "dynamical unlocking"
  - NAIC introduced in 2009 a new standard valuation law called "Principle-Based Reserving"
- Unlocking of assumptions may be permitted, provided justified.
- Actuary must continually evaluate historical experience and make needed adjustments.
- All part of a bigger project at the Goldenson Center on "Claims Tracking and Monitoring"

An illustration

Consider a fully discrete n-year term insurance policy issued to a life aged x with a death benefit of M and an annual level premium of P determined according to the equivalence principle.

At policy duration t, the prospective loss random variable is

$$L_t^P = \mathsf{PVFB}_t - \mathsf{PVFP}_t,$$

where for our policy, we have

$$\mathsf{PVFB}_t = M\, v^{K_{x+t}+1}\, I(K_{x+t} < n-t) \text{ and } \mathsf{PVFP}_t = P\, \ddot{a}_{\overline{\min(K_{x+t}+1, n-t)}|}.$$

The prospective reserve is

$$\mathsf{E}(L_t^P) = \mathsf{E}(\mathsf{PVFB}_t) - \mathsf{E}(\mathsf{PVFP}_t) = M\, A^{\,1}_{x+t:\overline{n-t}|} - P\, \ddot{a}_{x+t:\overline{n-t}|}$$

A straightforward algebraic manipulation leads us to:

$$\text{Retrospective Reserve} = P\, \frac{\ddot{a}_{x:\overline{t}|}}{{}_tE_x} - M\, \frac{A^{\,1}_{x:\overline{t}|}}{{}_tE_x}$$

Defining the retrospective loss random variable

For a policyholder age x, denote curtate future lifetime by K_x.

For $K_x < t$, the policyholder dies before reaching age $x+t$ and we define the retrospective loss r.v. to be

$$L_t^R = \frac{1}{{}_t p_x} \left[P \ddot{a}_{\overline{K_x+1}|} (1+i)^t - M(1+i)^{t-K_x-1} \right].$$

- First term: the accumulation at time t of all past premiums paid
- Second term: the accumulation of death benefit at time t.

Here we interpret L_t^R to be the realized gain at time t, per surviving policyholder, attributable to all those who died before valuation date.

For $K_x \geq t$, we define the retrospective loss as the constant

$$L_t^R = \frac{P \ddot{a}_{\overline{t}|} (1+i)^t}{{}_t p_x}.$$

The two expectations are equal

By expressing retrospective loss random variable more succinctly as

$$L_t^R = \frac{1}{{}_t p_x} \left[P(1+i)^t \, \ddot{a}_{\overline{\min(K_x+1,t)|}} - M(1+i)^{t-K_x-1} \cdot I(K_x < t) \right]$$

and using formulas from life contingencies, we can write

$$\mathsf{E}(L_t^R) = \frac{1}{{}_t E_x} \left(P \, \ddot{a}_{x:\overline{t|}} - M A^{\;1}_{x:\overline{t|}} \right).$$

Now using the actuarial equivalence principle, $P \, \ddot{a}_{x:\overline{n|}} = M A^{\;1}_{x:\overline{n|}}$, it is straightforward to show equivalence:

$$\mathsf{E}(L_t^R) = \mathsf{E}(L_t^P).$$

The two random variables are not equivalent

The distributions of the two random variables are very much different, so that, for example, their variances are not the same.

Notice that at policy issue, when at $t = 0$, we have $L_0^R = 0$ although

$$L_0^P = B\, v^{K_x+1}\, I(K_x < n) - P\, \ddot{a}_{\overline{\min(K_x+1,n)|}}$$

and is not necessarily always equal to zero.

In contrast, at policy maturity $t = n$, the prospective loss is $L_n^P = 0$ since there is no more future net liabilities. However, the retrospective loss random variable at policy maturity is

$$L_n^R = \frac{1}{{}_nE_x}\left[P\, \ddot{a}_{\overline{\min(K_x+1,n)|}} - B\, v^{K_x+1}\, I(K_x < n)\right]$$

which also is not necessarily equal to zero.

Interpreting the difference

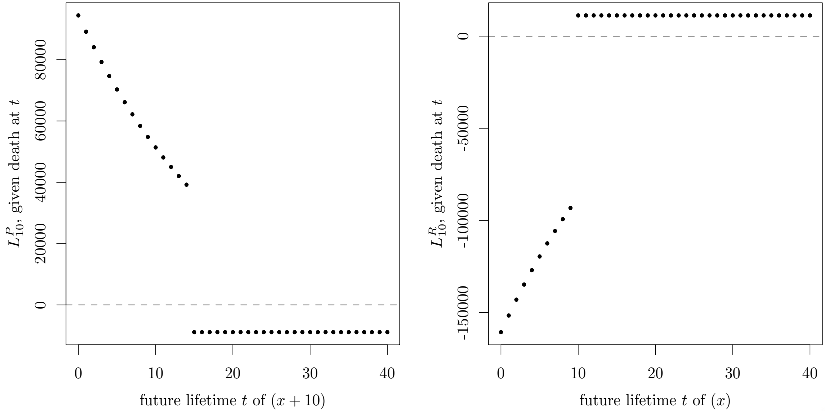
The prospective loss random variable can be viewed as:

"the share per survivor of the `present value of net liabilities` per \$1 of insurance at duration t"

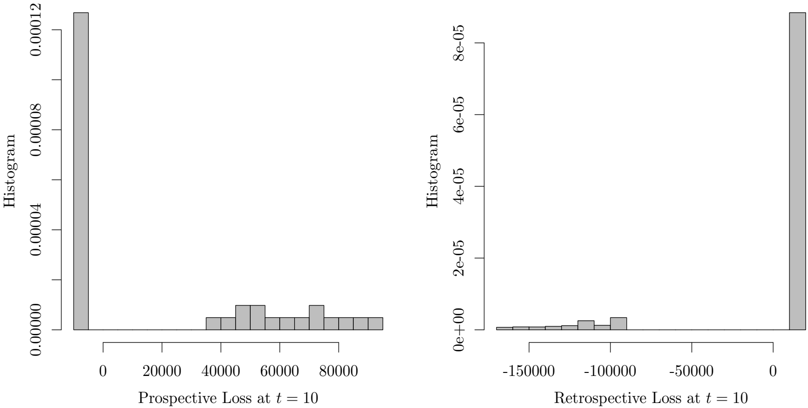
while the retrospective loss random variable as:

"the share per survivor of the `accumulated net assets` per \$1 of insurance at duration t."

Comparing realized prospective and retrospective loss at duration 10



Distribution of prospective and retrospective loss at duration 10



Numerical example

Duration	Retrospective Loss Random Variable		Prospective Loss Random Variable	
t	Mean	Standard Deviation	Mean	Standard Deviation
1	2.24	21.68	2.24	138.35
2	4.37	34.96	4.37	142.91
3	6.39	47.71	6.39	147.07
4	8.31	60.38	8.31	150.90
5	10.16	73.05	10.16	154.51
6	11.91	86.09	11.91	157.81
7	13.51	99.79	13.51	160.69
8	14.94	114.21	14.94	163.08
9	16.18	129.36	16.18	164.95
10	17.19	145.42	17.19	166.14
11	17.92	162.43	17.92	166.53
12	18.30	180.59	18.30	165.85
13	18.26	200.01	18.26	163.81
14	17.76	220.68	17.76	160.18
15	16.71	242.77	16.71	154.42
16	14.95	266.51	14.95	145.65
17	12.45	291.92	12.45	132.81
18	9.18	318.95	9.18	114.15
19	5.06	347.68	5.06	85.00
20	0.00	378.27	0.00	0.00

Table: Mean and standard deviation of retrospective and prospective loss random variables per $1,000, where $x = 45$, $n = 20$, $i = 5\%$, and Gender = Male.

Fully continuous whole life insurance

For a fully continuous whole life insurance, the retrospective loss is

$$L_t^R = \frac{1}{{}_tp_x}\left[\overline{P}\,(1+i)^t\,\overline{a}_{\overline{\min(T_x,t)}|} - M(1+i)^{t-T_x}\cdot I(T_x<t)\right]$$

The corresponding prospective loss is

$$L_t^P = M\,v^{T_{x+t}} - \overline{P}\,\overline{a}_{\overline{T_{x+t}}|} \tag{1}$$

Analogous to the fully discrete, the retrospective reserve is

$$\mathsf{E}(L_t^R) = \overline{P}\,\frac{\overline{a}_{x:\overline{t}|}}{{}_tE_x} - M\,\frac{\overline{A}^{\,1}_{x:\overline{t}|}}{{}_tE_x},$$

and the prospective reserve is

$$\mathsf{E}(L_t^P) = M\,\overline{A}_{x+t} - \overline{P}\,\overline{a}_{x+t}.$$

Using again the equivalence principle, $\overline{P}\,\overline{a}_x = M\overline{A}_x$, it follows that

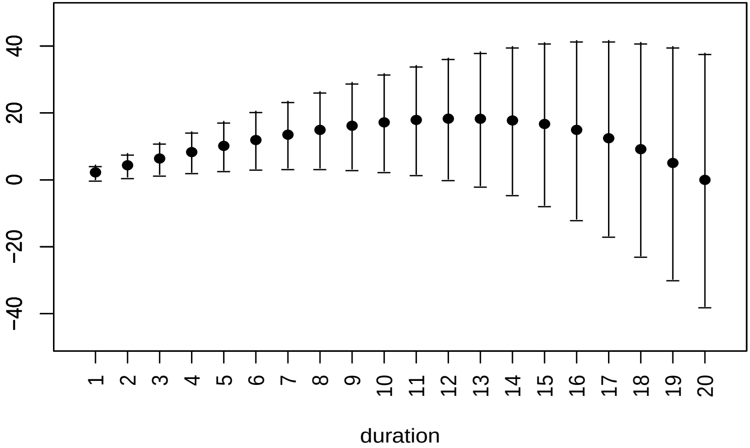
$$\mathsf{E}(L_t^R) = \mathsf{E}(L_t^P).$$

Portfolio of insurance policies

Duration	Mean	SD	Mean -0.1*SD	Mean +0.1*SD	Mean -0.2*SD	Mean +0.2*SD	Mean -0.5*SD	Mean +0.5*SD	Mean +SD	Mean -SD	Mean -3*SD	Mean +3*SD
1	2.24	2.17	2.03	2.46	1.81	2.68	1.16	3.33	0.08	4.41	-4.49	8.75
2	4.37	3.50	4.02	4.72	3.67	5.07	2.62	6.12	0.88	7.87	-8.74	14.86
3	6.39	4.77	5.91	6.87	5.43	7.34	4.00	8.77	1.62	11.16	-12.78	20.70
4	8.31	6.04	7.71	8.91	7.10	9.52	5.29	11.33	2.27	14.35	-16.62	26.42
5	10.16	7.30	9.43	10.89	8.70	11.62	6.51	13.81	2.86	17.47	-20.32	32.07
6	11.91	8.61	11.05	12.77	10.19	13.63	7.60	16.21	3.30	20.52	-23.82	37.74
7	13.51	9.98	12.51	14.51	11.51	15.51	8.52	18.50	3.53	23.49	-27.02	43.45
8	14.94	11.42	13.80	16.08	12.65	17.22	9.23	20.65	3.52	26.36	-29.87	49.20
9	16.18	12.94	14.89	17.48	13.60	18.77	9.71	22.65	3.25	29.12	-32.37	54.99
10	17.19	14.54	15.73	18.64	14.28	20.10	9.92	24.46	2.65	31.73	-34.38	60.81
11	17.92	16.24	16.30	19.55	14.67	21.17	9.80	26.04	1.68	34.17	-35.84	66.65
12	18.30	18.06	16.50	20.11	14.69	21.91	9.27	27.33	0.24	36.36	-36.60	72.48
13	18.26	20.00	16.26	20.26	14.26	22.26	8.26	28.26	-1.74	38.26	-36.51	78.26
14	17.76	22.07	15.55	19.96	13.34	22.17	6.72	28.79	-4.31	39.83	-35.51	83.96
15	16.71	24.28	14.28	19.13	11.85	21.56	4.57	28.84	-7.57	40.98	-33.41	89.54
16	14.95	26.65	12.29	17.62	9.62	20.28	1.63	28.28	-11.70	41.61	-29.91	94.91
17	12.45	29.19	9.53	15.37	6.61	18.29	-2.15	27.05	-16.74	41.64	-24.90	100.03
18	9.18	31.89	5.99	12.37	2.80	15.55	-6.77	25.12	-22.72	41.07	-18.35	104.86
19	5.06	34.77	1.59	8.54	-1.89	12.02	-12.32	22.45	-29.70	39.83	-10.13	109.37
20	0.00	37.83	-3.78	3.78	-7.57	7.57	-18.91	18.91	-37.83	37.83	0.00	113.48

Table: Mean, standard deviation and quantiles of the retrospective loss random variable per $1,000 for a portfolio, where $x = 45$, $n = 20$, $M = \$100,000$, $i = 5\%$, Gender = Male, and Number of Policies = 100.

Mean $\pm$ One Standard Deviation of the Retrospective Loss Random Variable



Implications of retrospective loss random variable

- Realized retrospective loss random variable represents historical claims experience.
- A significant deviation of the realized retrospective loss r.v. from the retrospective reserves could indicate that prospective reserves should be adjusted.
- Definition of significant deviation and level of adjustment of prospective reserves could use the GLM confidence approach (done in a separate work).
- A "credibility" adjustment could be created by having the confidence interval vary by reserve duration:
  - Later duration policies have tighter confidence intervals (CI's) since more historical experience available.

Credibility adjusted confidence intervals

- Overall consistency requirement is that the later the policy duration, the tighter the confidence interval because of more credible historical experience.
- Define the confidence interval width as $0.5 \times (\text{upper CI} - \text{lower CI})$:
  - Keep the confidence width fixed for each duration which leads to tighter CIs as duration increases since the SD of the retrospective reserve increases by duration.
  - Linearly decline the confidence width to zero from duration 1 to the end of the coverage period.
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Application: assumptions

- Hypothetical in-force block of 20 year, fully discrete term insurance policies issued over the past 10 years.
- For each issue year, 100 policies are issued and they are randomly issued over issue ages 35 to 55 and face amounts $100,000 to $500,000.
- Policy premiums are based on the equivalence principle.
- For durations 1 to 5 (i.e. more recent issues), actual historical mortality is assumed to be 25% lower than reserving assumptions.
- For durations 6 to 10 (i.e. earlier issues), actual historical mortality is assumed to be 25% higher than reserving assumptions.

- continued

- Prospective reserves are adjusted based on deviations of the realized retrospective loss random variable from the confidence interval of the retrospective loss random variable.
- The confidence interval is based on
  - $10\% \times$ SD for policies in duration 10 at the valuation date,
  - $20\% \times$ SD for policies in duration 9, etc. and
  - $100\% \times$ SD for policies in duration 1 at the valuation date.
- Assume that the only decrement is mortality and the prospective reserve is being calculated at end of duration 10.

Adjustment to prospective reserves

Duration	Issue Year	Realized Retro Loss RV Mean	Realized Retro Loss RV Deviation	Expected Prosp Loss Mean (Reserve)	Adjusted Prosp Loss Mean (Reserve)	Realized Prosp Loss Mean (Reserve)
10	1	13.30	-2.72	17.60	20.33	26.91
9	2	13.09	-0.60	16.60	17.20	26.04
8	3	13.00	Within Interval	15.86	15.86	25.68
7	4	11.07	Within Interval	13.10	13.10	22.10
6	5	11.31	Within Interval	13.05	13.05	23.05
5	6	12.44	Within Interval	11.20	11.20	1.12
4	7	7.94	Within Interval	7.26	7.26	-0.65
3	8	7.47	Within Interval	6.95	6.95	-2.54
2	9	4.55	Within Interval	4.32	4.32	-4.01
1	10	2.42	Within Interval	2.33	2.33	-6.13
	Aggregate	9.66		10.80	11.11	11.04

Table: Prospective reserve adjustment example

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Remaining Policies	993
Remaining Policies Face Amount	297,226,683
Expected Retrospective Reserve	10.80
Expected Prospective Reserve	10.80
Adjusted Prospective Reserve	11.11
Realized Prospective Reserve	11.04
Expected Aggregate Prospective Reserve	3,210,105
Adjusted Aggregate Prospective Reserve	3,303,576
Realized Aggregate Prospective Reserve	3,280,768
Per $1000 Difference Between Expected and Realized Prospective Reserves	-0.24
Per $1000 Difference Between Adjusted and Realized Prospective Reserves	0.08
Aggregate Difference Between Expected and Realized Prospective Reserves	(70,662)
Aggregate Difference Between Adjusted and Realized Prospective Reserves	22,808

Table: Difference between the adjusted and expected prospective reserves

Adjusting prospective reserves in practice

For a given inforce block of policies, a company should do the following at the valuation date:

- Break up the inforce block by plan of insurance
- For a given plan of insurance, break up the policies by duration
- For a given duration, determine number of policies issued
- Calculate the realized retrospective reserve at date of valuation
- Compare against the retrospective random variable CI for that duration and determine the adjusted prospective reserve
- Calculate the total adjusted prospective reserves for that duration
- Repeat the above for all durations and all plans of insurance

This research is part of the applied research work undertaken by the Goldenson Center for Actuarial Research at the University of Connecticut. The research philosophy of the center is to do cutting-edge academic research work with implementable solutions based on real-life issues faced by industry by using faculty members and graduate students in collaboration with industry professionals.