Classical Reserving - Double Chain Ladder and its Extensions

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Motivation

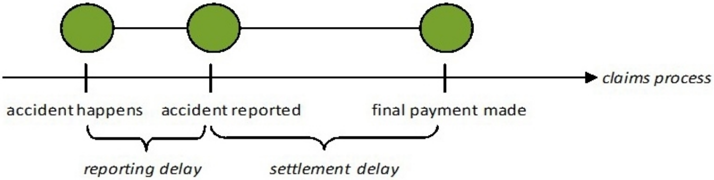
Non life insurance, reserve estimating outstanding liabilities:

- **Aim:** Forecast amount of claims which have been underwritten in the past, but are not settled yet.
  - Predict outstanding loss liabilities for debts to policyholders.
  - This is a significant part of the technical reserve in the financial statements of insurance companies.
  - This is an important point considering the sale of policies, because the outstanding liabilities have a large influence on the price.
  - **Solvency II:** EU directive which requires (among other things) several statistical quality standards in the models used to quantify the technical reserves.

Classical Reserving

The individual claims mechanism

The life of an individual claim in the general claims process:



Three categories of claim:

- Reported and paid
- Reported but not settled, RBNS
- Incurred but not reported, IBNR

The problem: stochastic reserving

Outstanding liabilities arise from two types of delay during the claims process:

- Reporting delay
- Settlement delay

Objectives:

- How large future claim payments are likely to be.
- The timing of future claim payments.
- The distribution of possible outcomes: future cash-flows.

Payment triangle

PAID	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	52	513	748	555	426	212	213	16	9	37	0	0	0	0	0	0	0	0	0	0
2	144	1006	910	736	593	766	615	245	116	15	36	165	15	67	6	11	0	0	0	
3	346	1467	1292	1237	1127	779	392	845	94	230	12	11	21	84	10	17	0	0		
4	408	1875	1810	1860	1806	1422	762	307	110	140	53	37	0	7	0	0	0			
5	712	3254	2696	2593	3377	2101	923	435	124	30	23	0	59	31	12	82				
6	941	3615	3274	4479	3841	2033	1242	472	120	59	5	0	9	0	0					
7	1221	5814	5905	7112	5321	2426	857	197	134	40	12	66	99	0						
8	1685	8164	7609	7722	6298	1981	830	580	198	124	64	29	48							
9	2253	9480	7697	8260	5872	2340	1099	363	147	44	14	19								
10	2043	8792	9169	7864	5895	1978	722	245	60	-1	34									
11	1570	9962	9670	8024	6121	2392	618	98	71	51										
12	1456	9182	8262	8374	4995	1886	883	241	64											
13	1129	7676	8515	6467	4505	1502	461	170												
14	1381	11548	8890	7964	4951	1980	475													
15	2196	12381	10391	7516	4969	1581														
16	2068	14179	11164	7740	4177															
17	1736	11607	8828	4883																
18	3269	15213	8372																	
19	4651	12172																		
20	4614																			

Figure: Motor Personal Injury payments

The Chain Ladder Method

- CLM is one of the most celebrated and well-known methods to estimate outstanding liabilities.
- developed in a time where closed form expressions where useful
- often gives reasonable results, intuitively appealing

The Chain Ladder Method:

Cumulative claims loss settlements	Development year							
Claims occurrence year	0	1	2	3	4	5	6	7
2005	1232	2178	2698	3420	3736	3901	3949	3963
2006	1469	2670	3378	4223	4684	4919	4975	
2007	1652	3068	4027	4981	5586	5873		
2008	1831	3465	4589	5676	6401			
2009	2074	3993	5323	6563				
2010	2434	4697	6358					
2011	2810	4918						
2012	3072							
CLM estimator for claims loss settlement factor		1,8508	1,3140	1,2422	1,1151	1,0491	1,0118	1,0035

3736+4684+5586+6401 = 20407

3420+4223+4981+5676 = 18300

20407/18300 = 1,1151

Figure: Source: Weindorfer (2012)

The Chain Ladder Method

Current method for calculating loss reserves: CLM

- simplicity and intuitive appeal
- operates from one run-off triangle (payments, incurred)

CLM suffers from these main drawbacks:

- originally just a clever algorithm, no statistical model
- Unstable estimates
- Unable to separate RBNS and IBNR claims

To address these limitations we introduce a statistical model.

Double Chain Ladder

The modelled data: two run-off triangles

- Incremental aggregated payment data.

SETTLEMENT →

ACCIDENT ↓

Payment data

i \ j	1	2	3	4	5	6	7
1	2200	1500	1000	650	300	150	100
2	1900	1400	900	550	250	145	
3	2300	1700	1200	750	400		
4	3000	1800	950	500			
5	2700	1500	1000				
6	3400	2200					
7	2500						

- Incremental aggregated counts data, which is assumed to be fully run off.

REPORTING →

ACCIDENT ↓

Counts data

i \ j	1	2	3	4	5	6	7
1	230	100	40	10	3	2	1
2	200	110	35	5	2	1	
3	210	85	25	7	2		
4	270	130	50	20			
5	240	100	45				
6	285	135					
7	240						

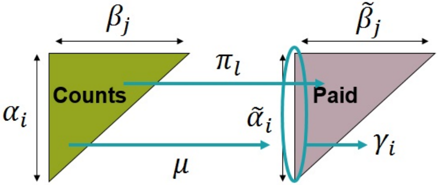
Double Chain Ladder

The data:

- **Aggregated payments:** $X_{\mathcal{I}} = \{X_{ij} : (i,j) \in \mathcal{I}\}$, with X_{ij} being the total payments from claims incurred in year i and paid with j periods delay from year i.
- **Aggregated incurred counts:** $N_{\mathcal{I}} = \{N_{ik} : (i,k) \in \mathcal{I}\}$, with N_{ik} being the total number of claims of insurance incurred in year i which have been reported with k periods delay from year i.

The DCL method to estimate the model

Double Chain Ladder is a statistical model which works on multiple run-off triangles.



The parameters involved in the model:

- Ultimate claim numbers: α_i
- Reporting delay: β_j
- Ultimate payment numbers: $\widetilde{\alpha}_i$
- Development delay: $\widetilde{\beta}_j$
- Individual payment mean in first period: μ
- Severity inflation: γ_i
- Settlement delay: π_l

First moment assumptions

M1 **The counts:**

- N_{ij} random variables with mean having multiplicative parametrization $E[N_{ij}] = \alpha_i \beta_j$
- identification $\sum_{j=0}^{m-1} \beta_j = 1$

M2 **The RBNS delay:**

- mean of RBNS delay variables is $E[N_{ijl}^{paid} | N_m] = N_{ij} \widetilde{\pi}_l$, for each $(i, j) \in I, l = 0, ..., m-1$.

M3 **The payments:**

- Conditional on the number of payments, the mean of the individual payments size is given by $E[Y_{ijl}^{(k)} | N_{ijl}^{paid}] = \widetilde{\mu}_l \gamma_i$.

Double Chain Ladder

Now, we can estimate our DCL parameters and obtain for the RBNS:

$$\widehat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} \widehat{\alpha}_i \widehat{\beta}_{j-l} \widehat{\pi}_l \widehat{\mu} \widehat{\gamma}_i.$$

We obtain for the IBNR:

$$\widehat{X}_{ij}^{ibnr} = \sum_{l=0}^{i-m+j-1} \widehat{\alpha}_i \widehat{\beta}_{j-l} \widehat{\pi}_l \widehat{\mu} \widehat{\gamma}_i.$$

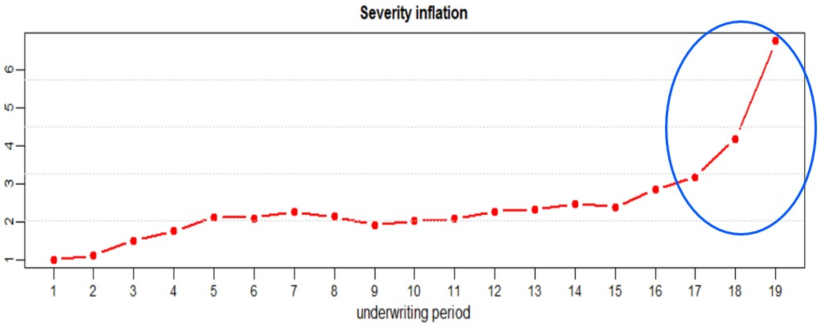
The outstanding loss liabilities point estimates are then

$$\widehat{X}_{ij} = \widehat{X}_{ij}^{rbns} + \widehat{X}_{ij}^{ibnr}.$$

Summary

- DCL produces (just like CLM) estimations for the total of the incremental payments.
- The classical chain ladder algorithm is applied twice to obtain estimates for all of the parameters in the model.
- They give the same value for the point estimates but DCL gives us more information.

The Double Chain Ladder model



The Double Chain Ladder model

Payments triangle

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19
1	51645	513057	747581	554656	426090	211996	212916	16199	9091	36933	0	0	0	0	0	0	0	0	0
2	143607	1006115	910371	735878	593070	765997	614642	245482	116065	14633	36298	164636	14601	66826	6415	11400	0	0	NA
3	345758	1467254	1291694	1236995	1127060	779156	391920	844780	94346	229871	12232	11210	20826	84329	9938	16898	0	NA	NA
4	408108	1875253	1809624	1859877	1806412	1422161	761701	306739	109769	139582	53448	36557	0	6731	0	0	NA	NA	NA
5	711788	3253701	2695979	2592550	3376797	2100946	923045	434936	124256	29942	23026	324	58834	31180	12306	NA	NA	NA	NA
6	941448	3614819	3273886	4479163	3841136	2032530	1241700	471996	120135	59047	5081	295	9393	0	NA	NA	NA	NA	NA
7	1221479	5814000	5904668	7112406	5320976	2425835	856998	196958	133568	40099	11797	65669	98728	NA	NA	NA	NA	NA	NA
8	1684782	8163947	7609088	7722323	6298256	1981161	830186	580355	197501	124446	63687	28557	NA	NA	NA	NA	NA	NA	NA
9	2253183	9479779	7696767	8260492	5871622	2339555	1099429	363351	147355	43520	13782	NA	NA	NA	NA	NA	NA	NA	NA
10	2042830	8791743	9169217	7864324	5894987	1977707	722425	245391	59786	-1390	NA	NA	NA	NA	NA	NA	NA	NA	NA
11	1570388	9961564	9669606	8024282	6120733	2391815	617560	97794	70961	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
12	1455847	9182448	8261734	8373519	4994670	1885764	882915	241387	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
13	1128853	7675536	8515497	6467241	4505204	1502376	460521	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
14	1380818	11547624	8890421	7964029	4951038	1980364	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
15	2195835	12381318	10390839	7516444	4968713	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
16	2068049	14178020	11164349	7740463	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
17	1747083	11599608	8808101	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
18	3294583	15210026	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
19	4664157	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

The reserve per underwriting year

	reserve	proportion of toal reserve
1	0.000000e+00	0.00
2	8.304134e+02	0.00
3	1.073025e+02	0.00
4	8.348906e+02	0.00
5	4.007342e+03	0.00
6	3.141223e+04	0.00
7	1.417988e+05	0.00
8	2.498179e+05	0.00
9	3.595187e+05	0.00
10	3.824873e+05	0.00
11	5.252174e+05	0.00
12	6.315314e+05	0.00
13	9.770538e+05	0.01
14	2.549259e+06	0.01
15	5.449377e+06	0.03
16	1.543851e+07	0.08
17	2.174178e+07	0.11
18	4.445951e+07	0.23
19	9.897470e+07	0.52

= 86% of total reserve

The Double Chain Ladder model

Summary of the major drawback of classical chain ladder (and thus the basic Double Chain Ladder method):

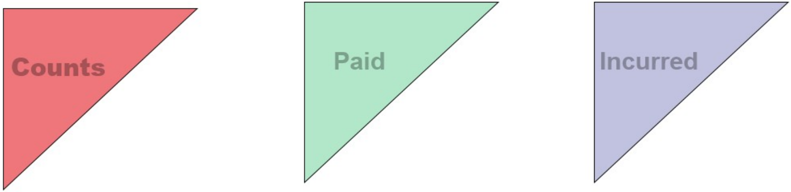
- The lack of sufficient data in the most recent underwriting years yields to a severity inflation estimation being too instable and thus not trustable in those most recent years.

- Even worse, those most recent underwriting years account for the very major part of the reserve.

Extensions of Double Chain Ladder

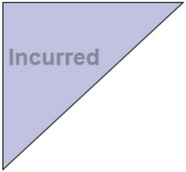
Extensions of Double Chain Ladder

Solution: Incorporate expert knowledge.



Extensions of Double Chain Ladder

The incurred triangle:



- It is not data, but a mixture of data and expert knowledge.
- It contains payments and case estimates of RBNS claims.
- From the incurred triangle, one can extract the RBNS part estimated by the case department.
- The RBNS case estimates differ from the DCL RBNS estimates.

Extensions of Double Chain Ladder

BDCL - Double Chain Ladder and Bornhutter-Ferguson:

- Aim: Adjust for unstable severity inflation.
- It shows that the severity inflation can be estimated from the incurred data.
- It replaces the fragile severity inflation coming from DCL by the more robust severity inflation of the incurred data.
- A number of empirical studies have shown that BDCL indeed is a robust and reliable reserving method.

Extensions of Double Chain Ladder

PDCL - RBNS Preserving Double Chain Ladder:

- Aim: Use expert knowledge and preserve RBNS case estimates in case of better representation of development pattern in incurred triangle.
- We are also able to adjust the RBNS case estimates if we think they are a bit over-/ underestimated.
- In PDCL, the RBNS reserve part is not just replaced by the case estimates.

Extensions of Double Chain Ladder

PDCL algorithm:

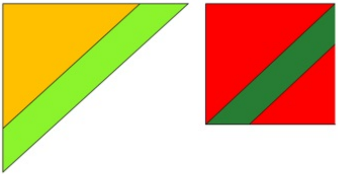
- Get RBNS and IBNR by DCL and replace RBNS with case estimates.
- Include IBNR and new RBNS in data and run DCL again to get new DCL parameters.
- These steps can be iterated until convergence.
- Since RBNS has changed as well, we correct the new RBNS result by changing the severity inflation so that we get the RBNS case estimates.
- Get new IBNR estimates with these new DCL parameters.
- Therefore, PDCL estimates the exact RBNS case estimates but also corrects the IBNR estimates.

Validation

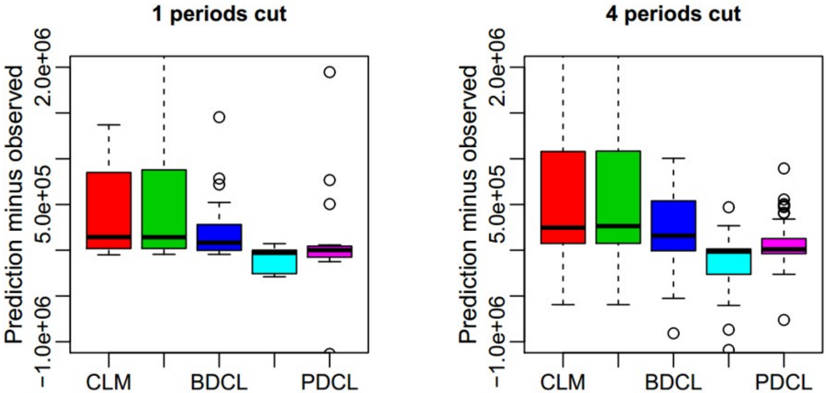
Validation

Testing results against experience:

- Cut c=1,2,... diagonals (periods) from the observed triangle.
- Apply the estimation methods.
- Compare forecasts and actual values.



Validation


1 periods cut
4 periods cut
Prediction minus observed
-1.0e+06
5.0e+05
2.0e+06
CLM
BDCL
PDCL

Next steps

- Include zero-claims probability.
- Include development year inflation.
  - very useful for split between big claims and small claims (for reinsurance purposes)
- Include options to adjust for data with special properties (e.g. strange numbers in one year because of financial crises).
- Operational time (time changes speed in every year).

England, P. and Verrall, R. (2002).
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