A systematic view on price based trading strategies

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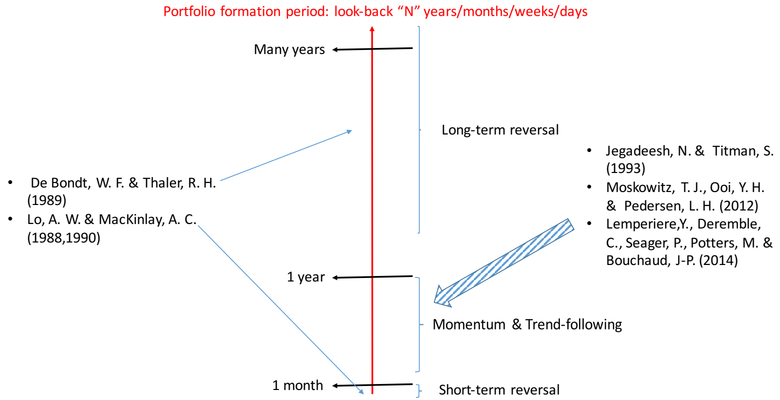
Introduction

Using past price to predict the price future has a well established history....



Contribution

- This study covers all portfolio formation periods which are traditionally analysed separately
- This study focus on individual assets and not on the cross-section (most literature). Closer in concept to Trend-Following (Lemperiere et al.) or Time-series momentum (Moskowitz et al.).
- Close form expression for both the return and the risk: Sharpe ratio shows that drift and autocorrelation have a different signature.

Contribution

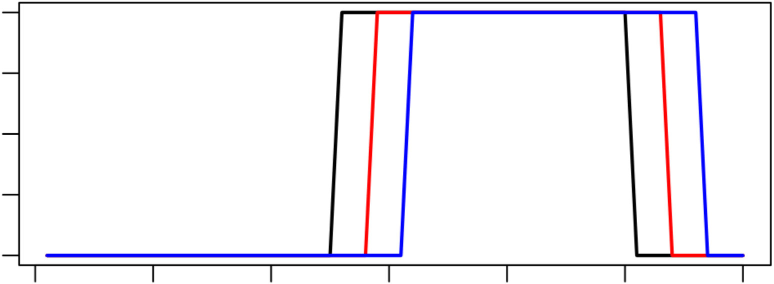
- Empirical study focuses on stock indices and contrasts stationary (in-sample) vs non-stationary (out-of-sample) strategy performance.
- We document long-term trend-following after a period of long-term reversal.
- We propose a non-stationary model that combines autocorrelation and periodic oscillations.

Our toy model/strategy

Moving average of past price returns:

$$\begin{cases} X_t = \ln(S_t/S_{t-1}), S_t = Price\ in\ period\ "t" \\ m_{t-1}(N) = \sum_{i=1}^{N} X_{t-i}/N \\ m_{t-1}(N) > 0 \to Buy\ \ m_{t-1}(N) \\ m_{t-1}(N) < 0 \to Short\ \ m_{t-1}(N) \end{cases} \tag{1}$$

Where $m_{t-1}(N)$ can be thought as a fraction of initial wealth to be invested in period t. Same proxy strategy used in several articles.



Return and Risk

Our moving average strategy can be solved in close form for both the mean $< R >$ and variance $< R^2 > - < R >^2$ as a function of the asset drift (μ), variance (V) and autocorrelation (ρ):

Average return of strategy:

$$\langle R \rangle = \left\langle X_t \overbrace{\frac{1}{N}\sum_{i=1}^{N} X_{t-i}}^{m_{t-1}(N)} \right\rangle = \frac{1}{N}\sum_{i=1}^{N} \langle X_t X_{t-i} \rangle = \mu^2 + \frac{V}{N}\sum_{i=1}^{N} \rho(t, t-i)$$

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Variance of strategy:

$$< R^2 > - < R >^2 = \frac{1}{N^2} \Bigg[N V^2 + N \mu^2 V + N^2 V \mu^2$$

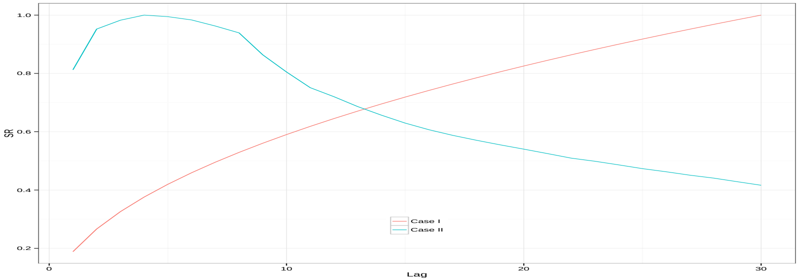
$$+ V^2 \Big(\sum_{i=1}^{N} \rho(t, t-i) \Big)^2 + V^2 \sum_{i,j=1, i \neq j}^{N} \rho(t-i, t-j)$$

$$+ \mu^2 V \left(2 \sum_{i=1}^{N} \rho(t, t-i) + \sum_{i,j=1, i \neq j}^{N} \Big(\rho(t, t-j) + \rho(t-i, t-j) + \rho(t, t-i) \Big) \right) \Bigg]$$

Sharpe Ratio

Case I: $\rho(t, t-i) = 0$, **Sharpe ratio is**

$$SR = \frac{\mu^2}{\sqrt{V\mu^2 + \frac{V^2}{N} + \frac{\mu^2 V}{N}}} \xrightarrow[N\to\infty]{take} \frac{|\mu|}{\sqrt{V}}$$



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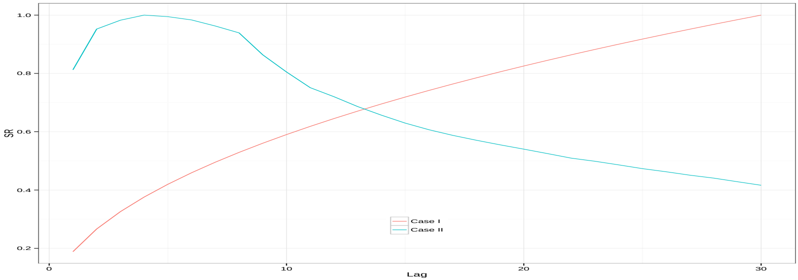


Case II: $\mu = 0$**, Sharpe ratio is**

$$SR = \frac{\sum_{i=1}^{N} \rho(t, t-i)}{\sqrt{N + (\sum_{i=1}^{N} \rho(t, t-i))^2 + (\sum_{i,j=1,i\neq j}^{N} \rho(t-j, t-i))}}$$

Empirical analysis

Data

- Start with daily prices of the DJIA (May, 1895 - Dec, 2015) and build weekly prices (S_t). There are going to be 5 time series: only Mondays all the way to only Fridays.
- Work with weekly log-returns $r_t = \ln(S_t/S_{t-1})$ but using linear-returns is similar.

Analysis

- In-sample & stationary: we perform simple data munging in order to conform with theoretical assumptions
- Out-of-sample & non-stationary: Data processing is minimal and conforms with back-testing paradigm. This is closer to the literature and also closer to real life implementation of our toy strategy. But by construction formula can only comply if auto correlation is very particular.

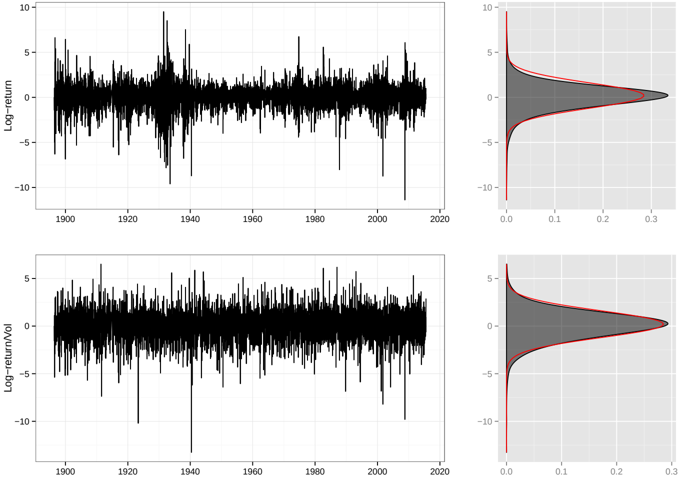
In-sample empirical analysis

We would like our returns to be stationary Gaussian random variables. That does not exist in real life ... These are the main issues we need to deal with:

- Volatility clustering and non-Gaussian returns
- Drift (first moment) is not constant: different economic cycles
- Autocorrelation of the returns if present might not be constant through history

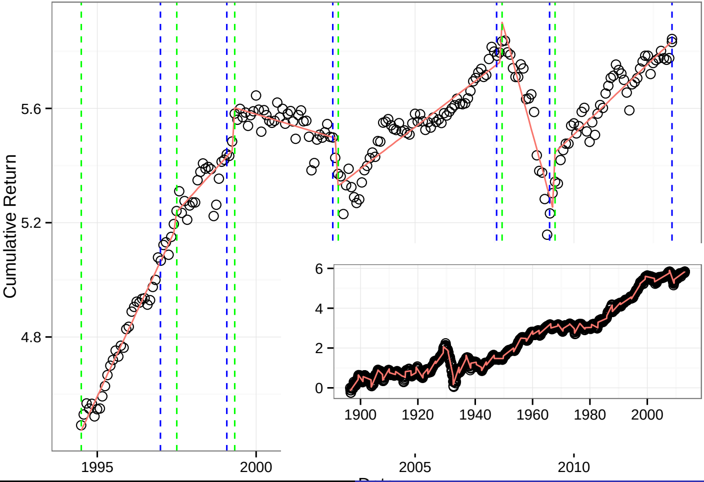
Volatility clustering: simple and approximate fix

Normalize the returns by a measure of local volatility: $X_t = \frac{r_t}{\sum_{i=1}^{i=p} |r_{t-i}/p|}$



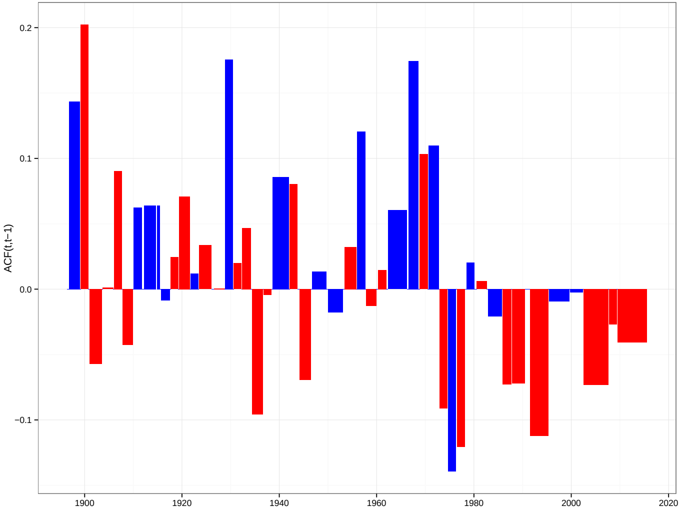
Drift changes: break up into approx constant drift periods

Different ways of finding stationary drift (macro, ad-hoc, statistical,...): we fit linear with a max of 69 break-points. We end up with 47 regimes between > 1.3 years and < 5 years (average 2.2 years).

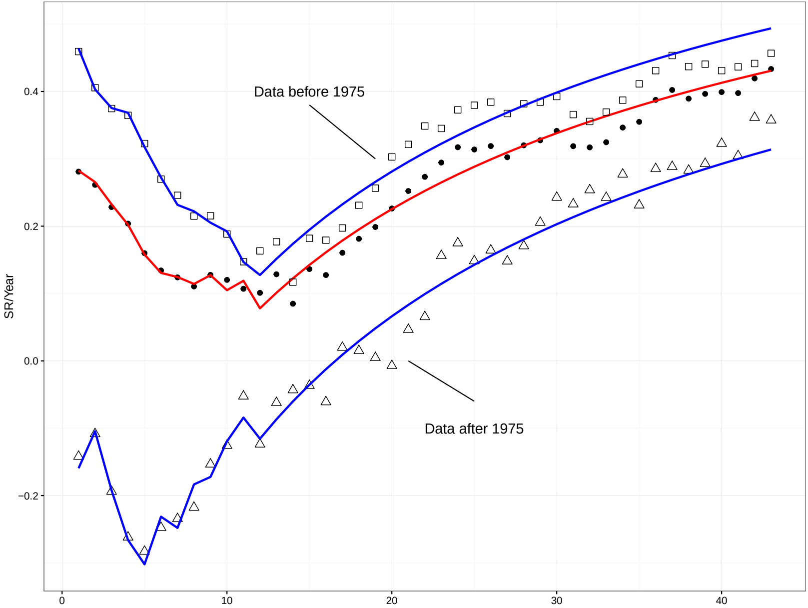


Autocorrelation: regime change

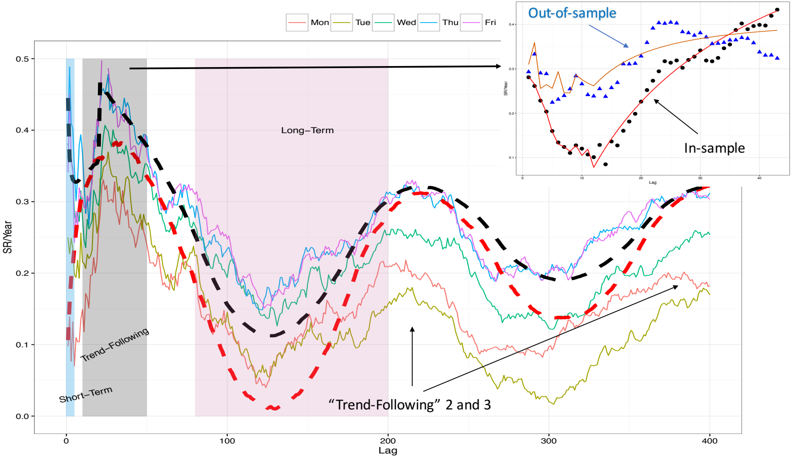
First lag auto-correlation $\rho(t, t-1)$ of weekly log-returns has a regime change around 1975.



Sharpe ratio for in-sample and stationary data



Out-of-sample empirical



Potential future research

- Same analysis for different technical rules.
- Same analysis for portfolio rules (cross-correlation between assets can be important).
- Understand transition from stationary to non-stationary: how exactly do the oscillations and auto-correlation appear?
- Try other algorithms to find stationary patches/regimes.
- Relax Gaussian condition in the calculation.

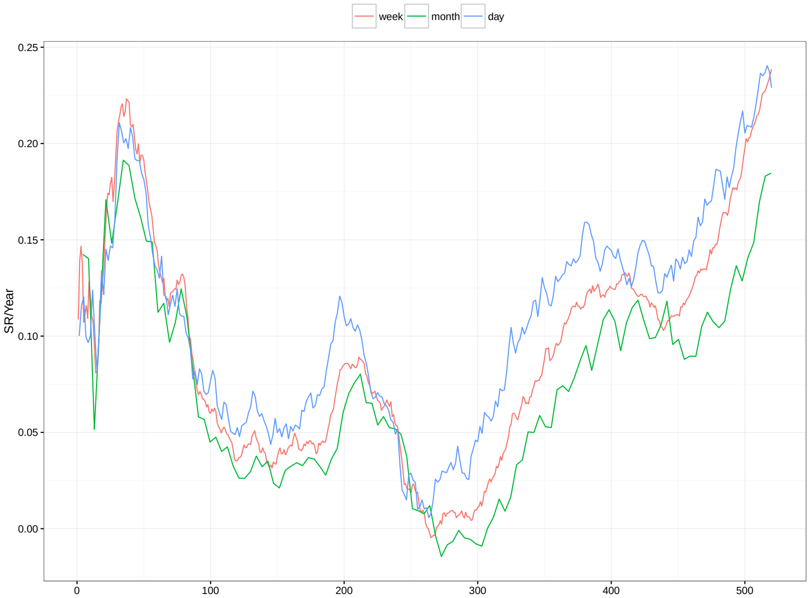
More information and sample code

- http://rpubs.com/silvaac/7420
- http://idatafactory.com

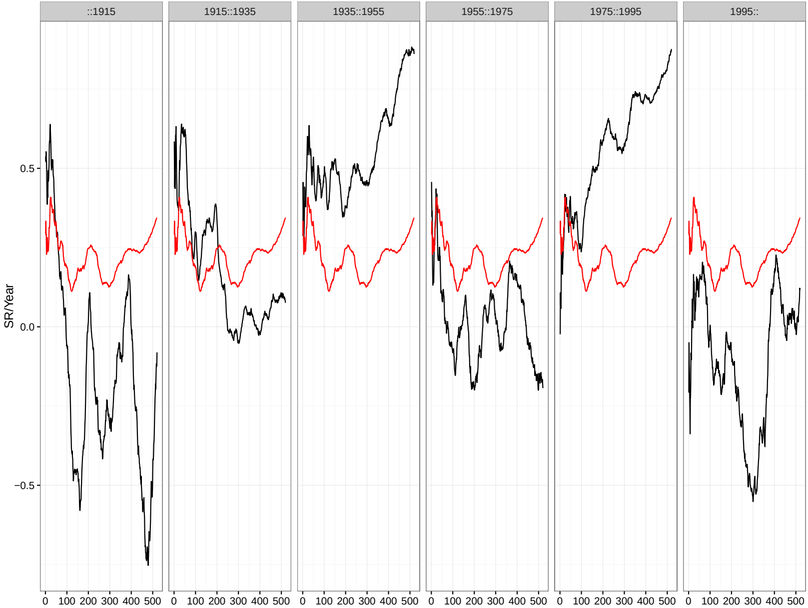
THANK YOU!

Out-of-sample: Effect of normalization (weekly data)

Out-of-sample: Effect of data freq (day,week or month)



Out-of-sample: 20 year sub-periods



Out-of-sample: More markets after 2004